
**Statistical methods for
implementation of Six Sigma —
Selected illustration of analysis of
variance**

*Méthodes statistiques pour la mise en œuvre du Six Sigma - Exemples
choisis d'application de l'analyse de la variance*

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Contents

Page

Foreword	iv
Introduction	v
1 Scope	1
2 Normative references	1
3 Terms and definitions	1
4 Symbols and abbreviated terms	4
5 General description of one-way and two-way classifications	4
5.1 General	4
5.2 Stating objectives	5
5.3 Data collection plan	6
5.4 Variables description	6
5.5 Measurement system considerations	6
5.6 Performing data collection	6
5.7 Verification of ANOVA assumptions	7
5.7.1 General	7
5.7.2 Test of normality	7
5.7.3 Test of homogeneity of variance	7
5.7.4 Test of independence	7
5.7.5 Outliers identification	7
5.7.6 How to deal with non-standard cases	8
5.8 Undertaking ANOVA analysis	8
5.8.1 State hypotheses H_0 and H_1	8
5.8.2 Graphical analysis	8
5.8.3 Generate analysis results	8
5.8.4 Residual analysis	8
5.9 Further analysis	9
5.10 Conclusion	9
6 Description of Annexes A through E	9
Annex A (informative) Bond strength	11
Annex B (informative) Effect of script and training on income per sale	19
Annex C (informative) Strength of welded joint	30
Annex D (informative) Water consumption in a petroleum enterprise	38
Annex E (informative) The hub total hours used on a task	45
Annex F (informative) ANOVA formulae	51
Bibliography	56

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT), see www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 7, *Applications of statistical and related techniques for the implementation of Six Sigma*.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

Analysis of variance (ANOVA) is a collection of statistical models used to analyse the differences among group means and their associated procedures (such as "variation" among and between groups), developed by statistician and evolutionary biologist Ronald A. Fisher. In the ANOVA setting, the observed variance in a particular variable is partitioned into components attributable to different sources of variation. In its simplest form, ANOVA provides a statistical test of whether or not the means of several groups are equal, and therefore generalizes the t-test to more than two groups. ANOVA models are useful for comparing (testing) three or more means (groups or variables) for statistical significance. It is conceptually similar to multiple two-sample t-tests, but is more conservative (it results in less type I error) and is therefore suited to a wide range of practical problems. In Six Sigma, ANOVA is used to find out if there are differences in the performances of different groups, and ultimately to find out if these differences count, or are important enough that a significant change or adjustment should be made. It serves as a guide on which aspect(s) of a process improvements can, or should, be made.

ANOVA is the synthesis of several ideas and it is used for multiple purposes. As a consequence, it is difficult to define concisely or precisely. Classical ANOVA for balanced data does the three following things at once.

- 1) As exploratory data analysis, an ANOVA is an organization of an additive data decomposition, and its sums of squares indicate the variance of each component of the decomposition (or, equivalently, each set of terms of a linear model).
- 2) Comparisons of mean squares, along with an F-test allow testing of a nested sequence of models.
- 3) Closely related to the ANOVA is a linear model fit with coefficient estimates and standard errors.

In short, ANOVA is a statistical tool used in several ways to develop and confirm an explanation for the observed data. Additionally:

- 1) it is computationally elegant and relatively robust against violations of its assumptions;
- 2) it provides industrial strength by (multiple sample comparison) statistical analysis;
- 3) it has been adapted to the analysis of a variety of experimental designs.

As a result, ANOVA has long enjoyed the status of being the most used (some would say abused) statistical technique in psychological research. "ANOVA" is probably the most useful technique in the field of statistical inference. ANOVA is difficult to teach, particularly for complex experiments, with split-plot designs being notorious.

There are three main assumptions:

- 1) independence of observations — this is an assumption of the model that simplifies the statistical analysis;
- 2) normality — the distributions of the residuals are normal;
- 3) equality (or "homogeneity") of variances, called homoscedasticity — the variance of data in groups is expected to be the same.

If the populations from which data to be analysed by a one-way analysis of variance (ANOVA) were sampled violate one or more of the one-way ANOVA test assumptions, the results of the analysis can be incorrect or misleading. For example, if the assumption of independence is violated, then the one-way ANOVA is simply not appropriate, although another test (perhaps a blocked one-way ANOVA) can be appropriate. If the assumption of normality is violated, or outliers are present, then the one-way ANOVA is not necessarily the most powerful test available. A nonparametric test or employing a transformation can result in a more powerful test. A potentially more damaging assumption violation occurs when the population variances are unequal, especially if the sample sizes are not approximately equal (unbalanced). Often, the effect of an assumption violation on the one-way ANOVA result depends on the extent of the violation (such as how unequal the population variances are, or how heavy-tailed one or

another population distribution is). Some small violations can have little practical effect on the analysis, while other violations can render the one-way ANOVA result uselessly incorrect or uninterpretable. In particular, small or unbalanced sample sizes can increase vulnerability to assumption violation.

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Statistical methods for implementation of Six Sigma — Selected illustration of analysis of variance

1 Scope

This document describes the necessary steps of the one-way and two-way analyses of variance (ANOVA) for fixed effect models in balanced design. Unbalanced design, random effects and nested design patterns are not included in this document.

This document provides examples to analyse the differences among group means by splitting the overall observed variance into different parts. Several illustrations from different fields with different emphasis suggest the procedure of the analysis of variance.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-1:2006, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

ISO 3534-3:2013, *Statistics — Vocabulary and symbols — Part 3: Design of experiments*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 3534-1, ISO 3534-3 and the following apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

- ISO Online browsing platform: available at <https://www.iso.org/obp>
- IEC Electropedia: available at <http://www.electropedia.org/>

3.1

response variable

variable representing the outcome of an experiment

[SOURCE: ISO 3534-3:2013, 3.1.3, modified — the notes have been removed.]

3.2

predictor variable

variable that can contribute to the explanation of the outcome of an experiment.

[SOURCE: ISO 3534-3:2013, 3.1.4, modified — the notes have been removed.]

3.3

model

formalized representation of outcomes of an experiment

[SOURCE: ISO 3534-3:2013, 3.1.2, modified — the notes and examples have been removed.]

3.4
analysis of variance
ANOVA

technique which subdivides the total variation of a response variable into components associated with defined sources of variation

[SOURCE: ISO 3534-3:2006, 3.3.8, modified — the notes and examples have been removed.]

3.5
degree of freedom
DF

number of linearly independent effects that can be estimated

[SOURCE: ISO 3534-3:2013, 3.1.32, modified — the symbol ν has been replaced with the abbreviated term DF, and the notes have been removed.]

3.6
factor

feature under examination as a potential cause of variation

[SOURCE: ISO 3534-3:2013, 3.1.5, modified — the notes have been removed.]

3.7
fixed effects analysis of variance

analysis of variance (3.4) in which the *factor levels* (3.8) of each *factor* (3.6) are preselected over the range of values of the factors

[SOURCE: ISO 3534-3:2013, 3.3.9, modified — the note has been removed.]

3.8
factor level

setting, value or assignment of a *factor* (3.6)

[SOURCE: ISO 3534-3:2013, 3.1.12, modified — the notes and the example have been removed.]

3.9
factor effect

factor (3.6) that influences the response variable

[SOURCE: ISO 3534-3:2013, 3.1.14, modified — the note has been removed.]

3.10
main effect

factor effect (3.9) applicable in the context of linearly structured *models* (3.3) with respect to expectation

Note 1 to entry: The main effect can be estimated by averaging the response variable over all other runs provided the experiment is fully balanced.

[SOURCE: ISO 3534-3:2013, 3.1.15, modified — Notes 1 and 3 have been removed; Note 2 has been renumbered as Note 1 to entry.]

3.11
one-way analysis of variance

analysis of variance (3.4) in which a single *factor* (3.6) is investigated

3.12
two-way analysis of variance

analysis of variance (3.4) in which two distinct *factors* (3.6) are simultaneously investigated for possible effects on the response variable

3.13**balanced data**

set of data in which sample sizes are kept equal for each treatment combination

3.14**F-test**

statistical test in which the test statistic has an F-distribution under the null hypothesis

3.15**p-value**

probability of observing the observed test statistic value or any other value at least as unfavourable to the null hypothesis

[SOURCE: ISO 3534-1:2006, 1.49, modified — the example and the notes have been removed.]

3.16**crossed classification**

classification according to more than one attribute at the same time

Note 1 to entry: Crossed classification can be illustrated in [Figure 1](#).

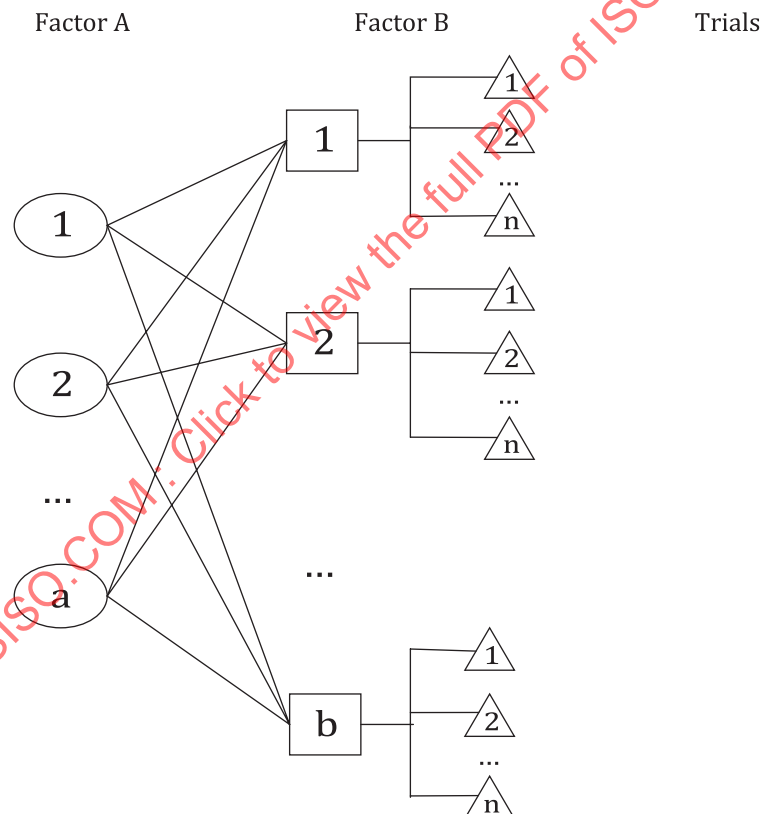


Figure 1 — Crossed classification graphic in ANOVA

3.17**interaction**

influence of one *factor* ([3.6](#)) on one or more other factors' impact on the response variable

[SOURCE: ISO 3534-3:2013, 3.1.17, modified — the notes have been removed.]

3.18 replication

multiple occurrences of a given treatment combination or setting of *predictor variables* ([3.2](#))

[SOURCE: ISO 3534-3:2013, 3.1.36, modified — the notes have been removed.]

4 Symbols and abbreviated terms

H_0	null hypothesis
H_1	alternative hypothesis
DF	degree of freedom
F	F-statistic
SS	sums of squares
MS	mean squares
Adj SS	adjusted sums of squares
Adj MS	adjusted mean squares

5 General description of one-way and two-way classifications

5.1 General

This clause provides general guidelines to conduct the one-way and two-way analysis of variances and illustrates the necessary steps. The formulae are shown in [Annex F](#).

Five distinct applications illustrating the procedures are given in [Annexes A](#) through [E](#). Each of these examples follows the basic structure in nine steps given in [Table 1](#).

The (common) flowchart for one-way and two-way ANOVA is given in [Figure 2](#).

Table 1 — General ANOVA procedure

1	Stating objectives
2	Data collection plan
3	Variables description
4	Measurement system considerations
5	Performing data collection
6	Verification of ANOVA assumptions
7	Undertaking ANOVA analysis
8	Further analysis
9	Conclusion

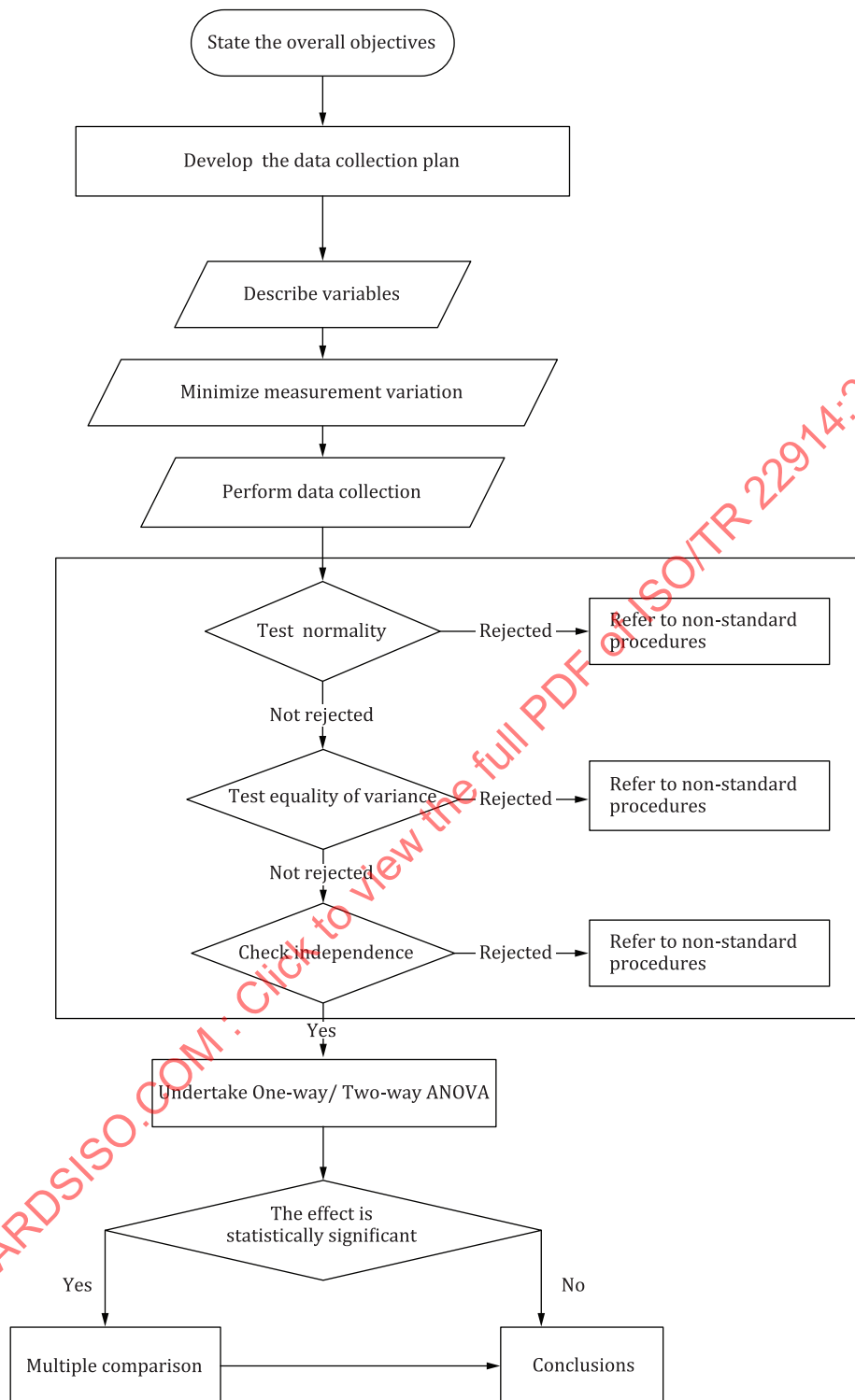


Figure 2 — Common flowchart for one-way and two-way ANOVA

5.2 Stating objectives

ANOVA is used to determine if there are differences in the mean in groups of continuous data. Analysis of variance is often used in Six Sigma projects in the 'analyse' phase of DMAIC (define, measure, analyse, improve, control) methodology. It is a statistical technique for analysing measurements depending on several kinds of effects operating simultaneously. Analysis of variance aims for deciding which kinds of

factors are important and estimating the effects of them. It is likely to be one of the most common tests that will be used by a Six Sigma project.

ANOVA is conducted for a variety of reasons, which include, but are not limited to:

- a) assess the need for a model to represent the data;
- b) test whether a factor with several levels is effective;
- c) test whether two factors have an interaction, which is only applicable for two-way ANOVA;
- d) test whether there is any difference between levels of some variables.

Analysis of variance examines the influence of one or two different categorical independent variables on one continuous dependent variable. One-way ANOVA examines the equality of the means of the continuous variable for each level of a single categorical explanatory variable. The two-way ANOVA not only aims at assessing the main effect of each independent variable but also if there is any interaction between them.

The analysis of variance can be presented in terms of a linear model. The objective of ANOVA is to find the differences between the data. It provides the basis for optimizing experiment design. Additionally, in Six Sigma, ANOVA is used to find out if there are differences in the performances of different factors. It serves as a guide on which aspect(s) of a process improvement can, or should, be made.

5.3 Data collection plan

The data collection plan describes the relationship with the design of the experiment; refer to Factsheet 26 in ISO 13053-2:2011^[1] for the design of the experiment. It contains the necessary steps for collecting, characterizing, categorizing, cleaning and contextualizing the data to enable its analysis.

The data collection plan also includes how to manage data quality. Data quality establishes the set of actions to be taken for ensuring the veracity of the data, such as integrity, completeness, timeliness and accuracy.

After collecting the data, it is highly recommended to check it for completeness (non-missing), errors or outliers, since these types of anomalies can distort the data.

For missing data, whether to use methods for dealing with missing data, such as imputation, or not is decided.

NOTE In statistics, imputation is the process of replacing missing data with substituted values. Once all missing values have been imputed, the data set can then be analysed using standard techniques for complete data. For more details about imputation methods, see Reference ^[2].

5.4 Variables description

Consists of describing the response variable and the independent factors and their relationship with the process.

5.5 Measurement system considerations

Consists of describing the measurement system analysis in place and the underlying requirements in order to minimize the measurement system variation. For details, refer to ISO 22514-6^[3] or ISO/TR 12888^[4].

5.6 Performing data collection

Consists of performing the data collection in accordance to the data collection plan in [5.3](#).

5.7 Verification of ANOVA assumptions

5.7.1 General

Analysis of variance is used to analyse the effects of factors, which can have an impact on the result of an experiment. This document focuses on fixed effects analysis of variance for data that satisfy three conditions: (1) the normality assumption; (2) the assumption of homogeneity of variances; (3) the independence of the observations.

5.7.2 Test of normality

There are two methods to test the normality of the model error: graphically and numerically, respectively relying on visual inspection and on statistics. In order to determine normality graphically, the output of normal probability plots, quantile-quantile plots (Q-Q plots) can be used. If the data are normally distributed, the Q-Q plot shows a diagonal line. If the Q-Q plot shows a line in an obvious non-linear fashion, the data are not normally distributed.

Numerically, the well-known tests of normality are the Kolmogorov-Smirnov test, the Shapiro-Wilk test, the Cramer-von Mises test and the Anderson-Darling test. They can be combined with the graphical analysis as performed in 5.8, refer to ISO 5479[5].

NOTE When the volume of data increases (which often happens nowadays), the coefficients of Pearson skewness and kurtosis can be taken into account. This is important because normality tests become powerful in this case and therefore rejects the hypothesis of normality simply for a small gap. However, the assumption of normality remains a good working hypothesis.

5.7.3 Test of homogeneity of variance

ANOVA requires that the variances of different populations be equal. This can be determined by the following approaches: comparison of graphs (box plots); comparison of variances, standard deviations.

The F-test of two sample hypothesis test of variances can be used to determine if the variances of two populations are equal.

5.7.4 Test of independence

ANOVA requires the independence of the observations. This can be determined, for example, by the following approaches:

- a) it can be checked by investigating the method of data collection; a pattern that is not random suggests a lack of independence;
- b) it can be evaluated by looking at the residuals against any time variables present (e.g., order of observation), any factors;
- c) it can be evaluated by looking at the auto-correlation and the Durbin-Watson statistic.

NOTE Data needs be sorted in correct order for meaningful results. For example, samples collected at the same time would be ordered by time if it is suspected that results could depend on time.

5.7.5 Outliers identification

For outliers' identification and treatment, refer to ISO 16269-4:2010[6] and ISO 5725-2:1994[7].

5.7.6 How to deal with non-standard cases

In many situations, the data do not fulfil all or part of the assumptions as described in 5.7.2 to 5.7.4. In these cases, the following several options can be adopted:

- transform the data using various algorithms so that the shape of the distribution becomes normally distributed;
- choose a nonparametric test, such as the Kruskal-Wallis H Test, which does not require the assumption of normality.

5.8 Undertaking ANOVA analysis

5.8.1 State hypotheses H_0 and H_1

State H_0 : the equality hypothesis among subgroups.

State H_1 : the inequality hypothesis among subgroups.

NOTE The hypotheses reflect the commonalities or the lack thereof among subgroups in business terms.

5.8.2 Graphical analysis

One can perform graphical analysis, i.e. histograms, box plots, to gain a better understanding of the data. Graphical analysis are linked to the business context and the data generating process.

5.8.3 Generate analysis results

A generic table for ANOVA is described, see Table 2.

Table 2 — Analysis of variance table

Variation	Cause	Source	Type	Sums of squares	Degrees of freedom	Variance estimate
Between	Assignable cause	Factor A	1-way and 2-way			
		Factor B	2-way			
		Interaction	2-way			
Within	Common cause	Error	1-way			
			2-way			
Total						

NOTE 1 The variance estimate is also known as mean squares.

NOTE 2 For the explicit formula in every case in Table 2 refer to Annex F. All the ANOVA tables can be interpreted in the same way. They allow to split the aggregate variability inside the data into two parts: assignable and common. The analysis of variance test determines whether the influence of assignable factors is statistically significant.

5.8.4 Residual analysis

Check residuals for independence, normality and auto-correlation using graphical visualisation or by quantitative methods. For graphical visualisation, it can be checked by residual plots. A residual plot is a graph that shows the residuals on one axis and the independent variable on the other axis.

The best test for auto-correlation is to look at a residual time series plot (residuals vs row number). If the plot of the residuals versus order does not show any pattern, there is no time dependence in the residuals.

The test for homogeneity of variance is to look at a plot of residuals versus predicted values. If the residuals are randomly scattered about zero and have approximately the same scatter for all fitted values, the constant variance assumption does not appear to be violated.

The best test for normally distributed errors is a normal probability plot or normal quantile plot of the residuals. If the points on the normal probability plot roughly follow a straight line, one can assume that the residuals do not deviate substantially from a normal distribution.

5.9 Further analysis

When a statistically significant effect in ANOVA exists, further analysis can be implemented. A statistically significant effect in ANOVA is often followed up with one or more different follow-up tests. This can be done in order to assess which groups are different from other groups.

Some tests such as Tukey's range test most commonly compare every group mean with every other group mean and typically incorporate some methods of controlling for Type I errors. Simple comparisons compare one group mean with one other group mean. Compound comparisons typically compare two sets of groups' means where one set has two or more groups (e.g., compare average group means of group A, B and C with group D).

For further analysis and model development, see References [8] and [9].

NOTE Tukey's range test, also known as the Tukey's test, Tukey method, Tukey's honest significance test, or Tukey's HSD (honestly significant difference) test, is a single-step multiple comparison procedure and statistical test. It can be used on raw data or in conjunction with ANOVA to find means that are significantly different from each other.

5.10 Conclusion

Based on the results of the above analysis of variance, some conclusions of the effect of the factor on the response variable can be obtained. With these findings, formulate a conclusion statement that links to the ANOVA results to the project objectives given in 5.2.

6 Description of Annexes A through E

Five distinct examples of ANOVA are illustrated in Annexes A to E, which have been summarized in Table 3 with the different aspects indicated.

Table 3 — Example summaries, by Annex

Annex	Example	ANOVA-details
A	Bond strength	Two-way ANOVA analysis: detects the factors which have effects on the bond strength. (Germany, Minitab ^{<?>} Minitab 17, R 3.0, JMP 11 and Q-DAS v12 are examples of suitable products available commercially. This information is given for the convenience of users of this document and does not constitute an endorsement by ISO of these products. 17)
B	Effect of script and training on income per sale	Two-way ANOVA analysis: detects the effect of script and training on income per sale (UK, R 3.0 ¹⁾)
C	Strength of welded joint	Two-way ANOVA analysis in DOE: detects the factors which have effects on the strength of welded joint. (India, JMP 11 ¹⁾)

Table 3 (continued)

Annex	Example	ANOVA-details
D	Water consumption in Petroleum enterprise	Two-way ANOVA analysis: tests whether there is significant difference among different teams and shifts. (China, Q-DAS V12 ¹⁾)
E	The hub total hours used on a task	One-way ANOVA analysis: detects whether the hours used on a task varied significantly by day of the weeks. (UK, Minitab 17 ¹⁾)

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Annex A (informative)

Bond strength

A.1 Stating objectives

An engineer, acting as a Six Sigma Green Belt, is planning to study the force required to separate two components held together by an adhesive bonding agent, to find the strongest adhesive formulation. The average pull force of the strongest adhesive formulation must be at least 50 daN.

The experimenter has assured that there are not any uncontrolled variables in the environment or been introduced in the experiment sequence. This is a general methodological requirement that needs to be verified prior to any analysis.

NOTE The Six Sigma Green Belt is looking to a pull test of strength. Twisting is excluded, and the engineer is not interested in any strength such as thermal shock, and there is no other catalyst in the environment or acid in the air. The engineer is trying to reproduce the conditions of use and not the condition of production.

A.2 Data collection plan

The engineer is considering three adhesive formulations: A, B and C. Each adhesive's ability to maintain strength over time is important. Using each formulation, the engineer prepares 72 assemblies so that 24 samples of each formulation can be pull-tested every 3 months to assess strength degradation. The 24 samples are collected in a random fashion.

A.3 Variables description

The considered response variable is the force required to separate two components held together. The unit of force is daN. For the sake of illustration, only pull force is considered.

There are two factors:

Factor 1: adhesive formulation, which has three levels.

Factor 2: lapsed time under extreme conditions (high temperature and humidity), which also has three levels: the third month, the sixth month, the ninth month (with a month being defined as a period of four weeks), because of the length of the period there might be massive change.

A.4 Measurement system considerations

Measurement error is small enough to be ignored.

A.5 Performing data collection

[Table A.1](#) provides the raw data used in the analysis of variance.

Table A.1 — Raw data

Formulation A			Formulation B			Formulation C		
Three months	Six months	Nine months	Three months	Six months	Nine months	Three months	Six months	Nine months
40,8	43,3	38,6	49,4	54,2	44,0	68,5	69,6	65,7
49,0	40,7	36,6	55,3	52,2	38,8	71,1	62,5	71,7
43,4	39,6	28,1	53,4	51,4	51,3	69,1	67,5	69,9
46,4	44,0	33,6	53,2	55,4	48,0	71,9	73,1	65,2
42,6	41,3	39,3	49,6	53,5	38,4	70,1	69,4	68,1
43,5	45,5	36,9	58,1	53,7	44,9	60,9	71,5	70,7
45,4	41,4	34,9	47,6	48,4	46,1	71,3	64,0	66,2
48,4	44,8	33,3	61,9	53,6	50,2	62,1	64,4	70,7
40,3	43,2	35,1	53,5	51,5	43,1	64,6	71,5	72,4
44,2	41,3	31,5	51,7	50,1	38,6	67,6	60,1	69,8
46,5	41,7	37,5	50,5	45,5	44,3	71,3	62,5	70,8
45,6	37,2	32,5	43,0	47,0	43,6	62,9	60,2	64,6
38,4	47,3	26,8	46,2	43,3	44,6	73,1	62,3	63,1
46,8	45,9	44,9	49,6	49,0	45,3	71,9	72,2	66,8
46,8	38,4	25,9	57,5	48,6	44,4	65,7	74,6	61,2
40,6	36,4	37,4	58,4	45,2	40,8	63,6	63,3	67,5
46,5	40,6	32,8	50,4	47,1	43,4	66,5	65,9	67,1
43,1	36,9	27,0	46,8	45,4	49,8	64,0	72,5	62,0
44,3	32,6	27,5	53,5	44,5	49,1	69,1	67,6	67,7
41,8	35,1	39,7	52,8	45,6	45,2	63,7	68,1	64,0
47,2	38,4	34,1	59,6	47,9	45,3	73,3	70,9	66,1
40,5	45,1	26,7	62,4	40,7	49,8	65,9	61,4	62,6
42,7	35,2	31,5	48,9	48,9	48,3	69,9	68,6	63,7
41,3	39,3	29,4	59,5	47,1	45,1	56,2	66,2	69,0

A.6 Verification of ANOVA assumptions

A.6.1 General

Verification of ANOVA assumptions mainly includes the aspects of normality, independence, homogeneity.

A.6.2 Test of independence

Independence can be checked by investigating the method of data collection. The experimenter has considered the use of randomization for collecting each 24 of the 216 samples.

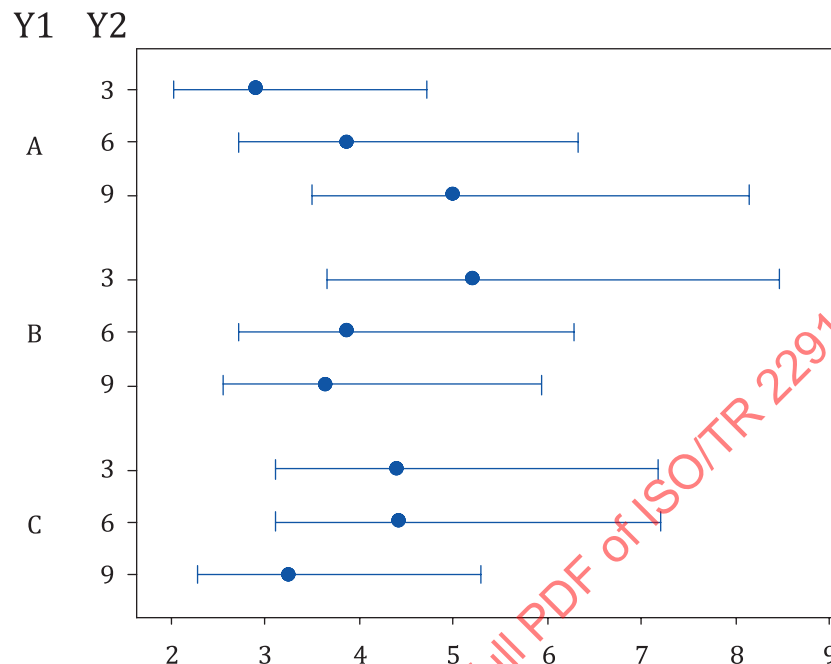
A.6.3 Test of normality

The normality of data is validated using the normal probability plot of residuals in a residual analysis procedure (see [Figure A.3](#)).

The probability plot shows that normal assumption is satisfied.

A.6.4 Test for homogeneity of variance

Next, the equivalence of variance is tested in 9 samples with 24 observations each. Minitab¹⁾ creates [Figure A.1](#).



Key

Y1 formulation

Y2 month

The Bartlett's test p -value is 0,111.

Figure A.1 — 95 % Bonferroni confidence intervals

[Figure A.1](#) shows the 95 % Bonferroni confidence intervals for standard deviations and the p -value of Bartlett's test. The 95 % Bonferroni confidence intervals arise from a multiple comparison test. If two intervals do not overlap, the difference between the corresponding standard deviations is statistically significant. From [Figure A.1](#), it can be seen that all of these intervals overlap. Moreover, the p -value of Bartlett's test is 0,111, which is greater than 0,05, suggest that there is no evidence to reject the equality of variances.

NOTE In statistics, Bartlett's test (see Reference [10]) is used to test if k samples are from populations with equal variances.

A.7 Undertaking ANOVA analysis

The hypotheses H_0 and H_1 are as follows:

H_0 : there is no difference in the pull force among different formulations and months.

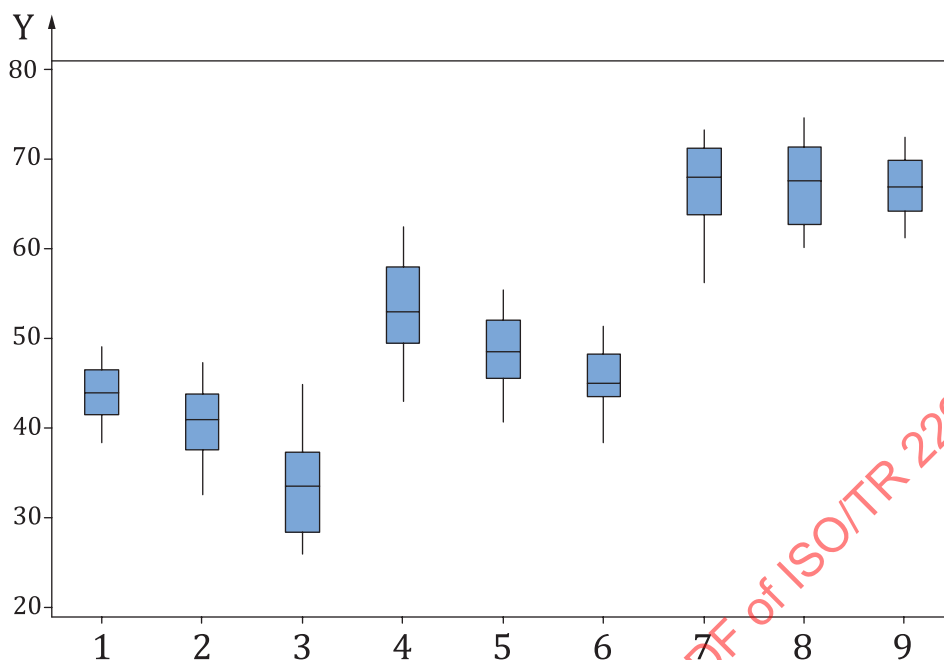
H_1 : there is a difference in the pull force between at least one formulation and/or one month and the others.

Then, a graphical analysis is conducted to know more about data quality.

Graphical analysis

1) Minitab is an example of a suitable product available commercially. This information is given for the convenience of users of this document and does not constitute an endorsement by ISO of this product.

A box plot of the data is displayed for each formulation and month to assess the mean, range, and shape of the data, see [Figure A.2](#).



Key

Y	force	4	three-B	7	three-C
1	three-A	5	six-B	8	six-C
2	six-A	6	nine-B	9	nine-C
3	nine-A				

Figure A.2 — Box plot of pull force

Minitab creates each box and whisker (the line extending from each end of the box) based on the spread of the data. The within-group variability is assessed by looking at the spread in the interquartile range, whiskers, and outliers. The variability between formulations and bonding time is assessed by comparing the medians and the relative locations of the interquartile ranges.

The box plots show that:

- at each bonding time, formulation A has the lowest pull force, formulation B has the second lowest pull force, and formulation C has the highest pull force;
- the pull force from formulation C is constant over the bonding times.

ANOVA procedure

The full model is analysed, which contains the two main effects and their interaction. The factors and the response for the model are entered. In this example, there are two factors: formulation and month. [Table A.2](#) shows the factor information. The response is the variable of interest — in this case, the force required to separate components. Minitab creates the result of [Table A.3](#).

Table A.2 — Factor information

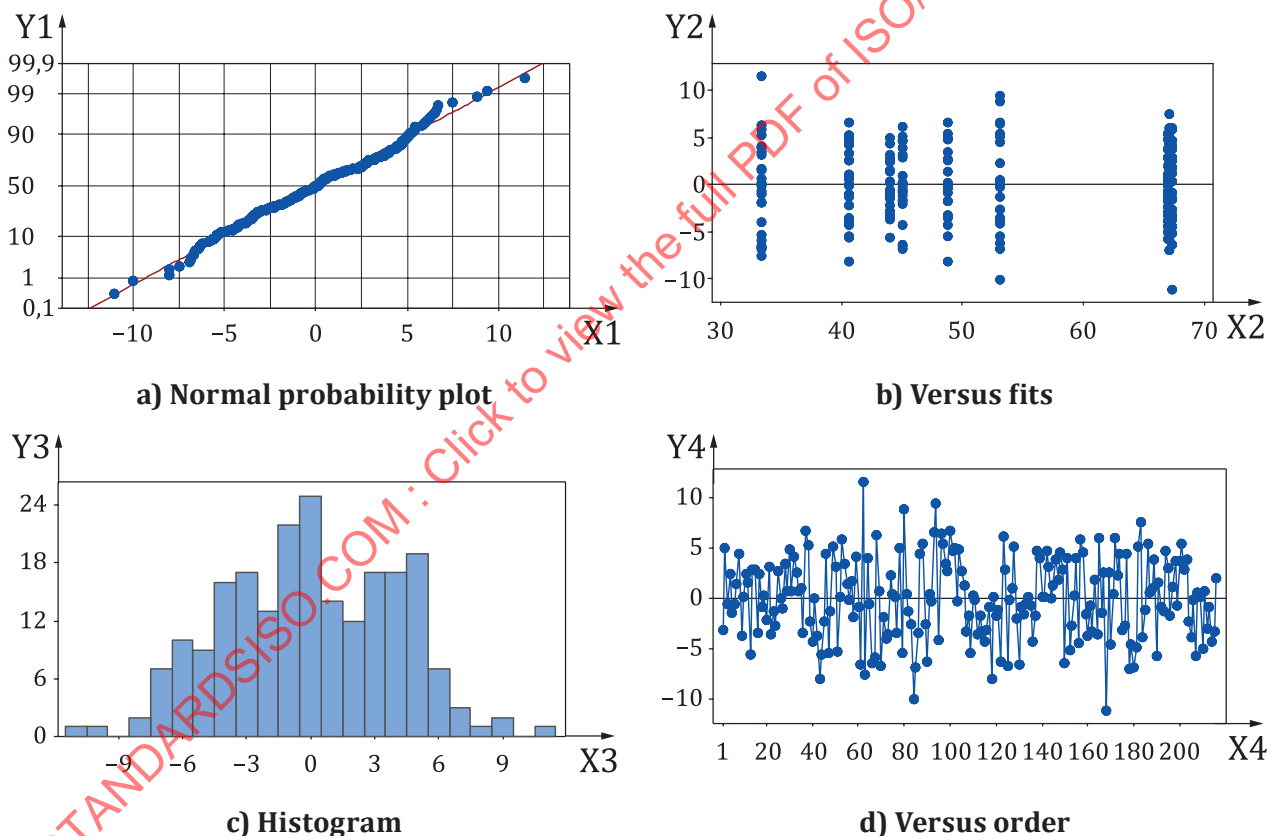
Factor	Type	Levels	Values
Formulation	Fixed	3	A,B,C
Month	Fixed	3	3,6,9

Table A.3 — Result of ANOVA: pull force versus formulation, month

Source	DF	Adj SS	Adj MS	F-value	p-value
Month	2	1 435,9	718	42,68	0,000
Formulation	2	28 591,3	14 295,6	849,89	0,000
Formulation × month	4	731,3	182,8	10,87	0,000
Error	207	3 481,9	16,8		
Total	215	3 4240,4			
R-square: 89,83 %		R-square (adjusted): 89,44 %			

The p -value are used to test the significance of each term. In this model, all of the terms are significant at the 0,05 α -level. A significant two-factor interaction indicates that the effect of a factor on the response depends on the level of the other factor. In this example, the effect of bonding time on pull force depends on the formulation used. Thus, it does not make sense to individually interpret the effects of month and formulation. The model explains 89,83 % of the variability in the response.

Residual analysis



Key

X1	residual	X2	fitted value	X3	residual	X4	observation order
Y1	percent	Y2	residual	Y3	frequency	Y4	residual

Figure A.3 — Residual plots for pull force

The residual plots are used to check the assumptions about the error distribution again. [Figure A.3](#) is the four-in-one residual plot.

- **Normal probability plot** – Because the points on the normal probability plot roughly follow a straight line, one can assume that the residuals do not deviate substantially from a normal distribution.

- **Histogram** –the normal probability plot is used to make a decision about the normality of the residuals. With a reasonably large sample size, the histogram displays compatible information.
- **Versus fits** – The constant variance assumption does not appear to be violated because the residuals are randomly scattered about zero and have approximately the same scatter for all fitted values.
- **Versus order** – The plot of the residuals versus order does not show any pattern. Therefore, there is no time dependence in the residuals.

Understanding the effects

Minitab provides two plots to help visualize the effects.

- Main effects plots show the relative strength and the direction of the effects. [Figure A.4](#) is the main effects plot for pull force.
- Interaction plots show how the level of one factor influences the effect of another factor. [Figure A.5](#) is the interaction plot for pull force. Because the interaction between formulation and month is significant, only the interaction plot is displayed.

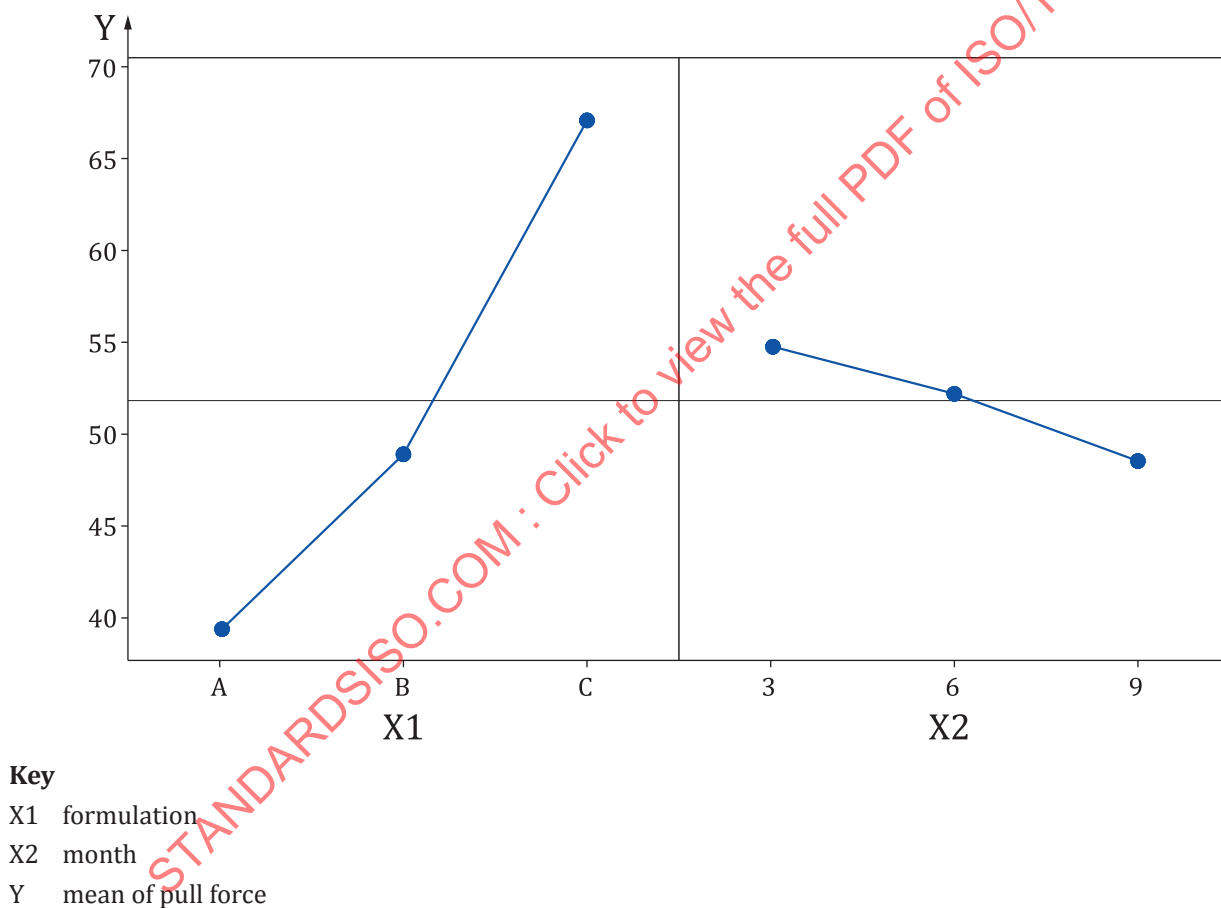
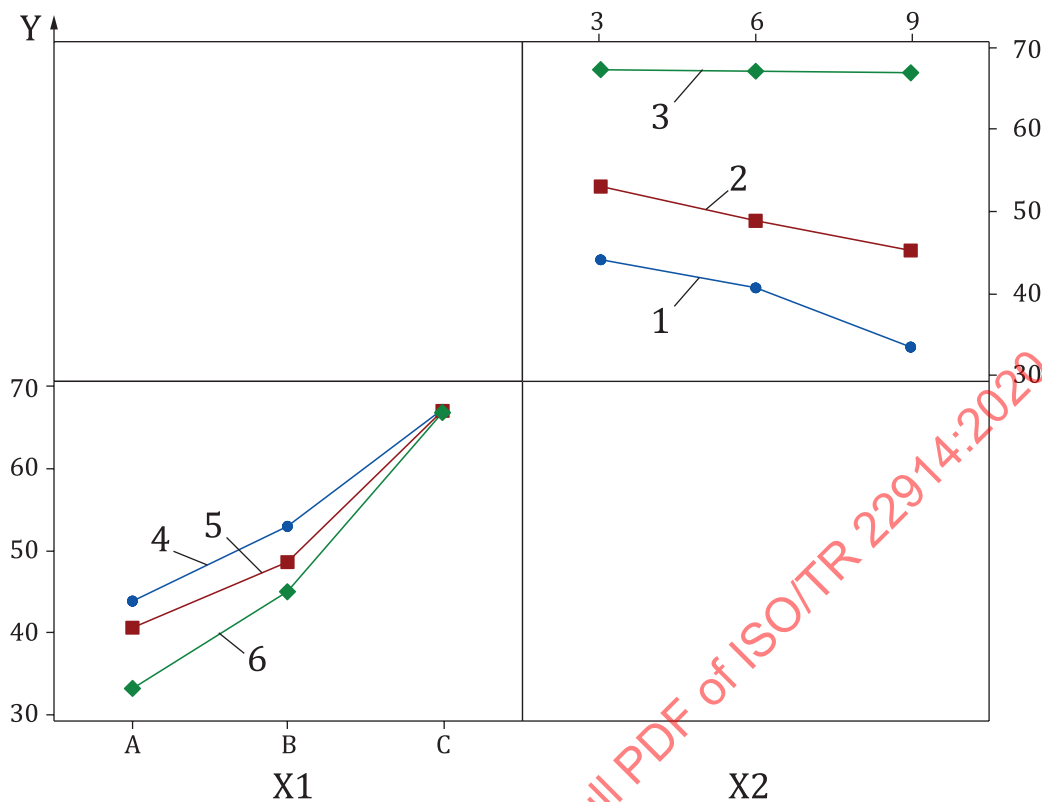


Figure A.4 — Main effects plot for pull force



Key

X1	formulation	1	formulation A	4	month 3
X2	month	2	formulation B	5	month 6
Y	force	3	formulation C	6	month 9

Figure A.5 — Interaction plot for pull force

The interaction plot shows how the level of one factor influences the effect of another factor. The lines on the plot are not parallel, which suggest there is an interaction. The *p*-value in the ANOVA table is 0,000 which shows that the interaction is significant.

This plot also shows that, on average:

- formulation C has the highest pull force;
- there seems to be no difference between the bonding times for formulation C;
- the pull force decreases over time for both formulations A and B.

A.8 Further analysis

Multiple comparisons

The *p*-values in the ANOVA table ([Table A.3](#)) indicate differences between the factor level means, but do not identify the means that differ. To determine which mean differences are significant, one can test them using one of the comparison methods.

Tukey's method compares all possible pairs of level means of the specified factors and interaction terms. [Table A.4](#) is the group information table.

Table A.4 — Group information table

Formulation*Month	N	Mean	Grouping					
Three-C	24	67,262 5	D					
Six-C	24	67,075 0	D					
Nine-C	24	66,891 7	D					
Three-B	24	53,033 3		E				
Six-B	24	48,741 7			F			
Nine-B	24	45,100 0			F	G		
Three-A	24	44,004 2				G	H	
Six-A	24	40,633 3					H	
Nine-A	24	33,400 0						J

NOTE Means that do not share a letter are significantly different.

The grouping information table shows, in a summarized format, factor level means that are significantly different based on the multiple comparison tests. Groups that do not share a letter are significantly different. Groups that share a letter are not significantly different. Using $\alpha = 0,05$ for all tests, the results indicate the following.

- The three means for formulation C are not statistically different from one another, but are statistically higher than all other combinations. They also meet the goal of an average pull force at least 50 daN.
- For formulation B, the three month pull force is statistically different from the six and nine month as well as the pull forces for all bonding times of formulation A.
- The three month pull force for formulation A is not statistically different from the six month pull force or the nine month pull force for formulation B.
- The nine month pull force for formulation A is statistically worse than all other combinations of formulation and bonding time.

A.9 Conclusion

The project objective is to find the strongest adhesive formulation. Based on the ANOVA analysis of the pull force, it can be concluded that:

- the best formulation is C and it meets the average pull force requirement;
- formulation C has a consistent pull force across the bonding times;
- formulations A and B have a decreasing pull force over the bonding time.

Annex B (informative)

Effect of script and training on income per sale

B.1 Stating objectives

The sales department of a car insurance company wants to find the key factors that affect the income for each sales agent. This department handles inbound phone calls, generated through marketing and advertising. The analysis of the people's performance in the department had identified a high variation in the outcome of the sales calls. Through a facilitated root cause problem solving session, two key opportunities arose:

- 1) the type of training which the agents went through, included the following:
 - a) soft skills (how to build rapport with customers), and
 - b) objection handling (understanding the specific objections that a customer could have with the price of the insurance product);
- 2) the script which they are using; these are specific text which the agent has to read out, word for word, usually to ensure the company is consistent and compliant with regulations.

The sales contact centre has hypothesised that both the training provided to staff, and the script they use, have an impact on the income per sale.

The example given relates to the improve phase of a DMAIC (define, measure, analyse, improve, control) project within the sales department of a car insurance company.

B.2 Data collection plan

Twenty agents are selected for the test and split into four groups of five. The twenty agents have the same demographic of experience and the call types (i.e. customer requirements) are just the calls asking for the same service. Each group of participants is proportionally demographic considering level of experience and type of shift. A random selection of 40 calls are taken for each team (8 per agent) over a two-week period.

Group 1 is trained in soft skills and uses the old (original) script.

Group 2 is trained in objection handling and uses the old (original) script.

Group 3 is trained in soft skills and uses the new script.

Group 4 is trained in objection handling and uses the new script.

There is only one type of call received in this contact centre, which is the sales call.

B.3 Variables description

The considered response variable is income per sale.

There are two factors, both having two levels, as follows.

Factor A is different scripts for the call centre agent: new versus old. The old script has been used for several years, however there is a hypothesis that customers now prefer a more flowing conversation,

with less structured questions. The new script allows the customers to provide the information in whichever order they would prefer. Both ensure the conversations fall within the required compliance.

Factor B is different training modules: soft skills training versus objection handling. Soft skills training focuses on the conversation skills of the agent, to engage in a personal albeit professional conversation with the customer, whereas the objection handling training gives the agent specific suggestions on what to say to the customer in response to specific objections to the sale raised by the customer.

B.4 Measurement system considerations

Outputs are measured by an integrated system which connects the telephone and user operational system. The telephone system details the number of calls received and the operational system generates the sale price, net of any discount given by the agent. Measurement of income per sale is automatically calculated by the IT system which the agents used. For each sale it considered the total price paid (Premium), less cost to the underwriter and also any discount which has been applied by the agent. Verification is completed by manually checking a sample. There is no need for a standard operating procedure (SOP) to ensure consistency in measurement as it is completely automated (i.e. same system is used for online quotes which can exceed 1M per day).

B.5 Performing data collection

[Table B.1](#) provides the part of the raw data used in the analysis of variance.

Table B.1 — Raw data

Income per sale	Script	Training
35,5	Original script	Soft skilled trained
31,5	Original script	Soft skilled trained
34,2	Original script	Soft skilled trained
34,4	Original script	Soft skilled trained
35	Original script	Soft skilled trained
27	Original script	Soft skilled trained
31	Original script	Soft skilled trained
35	Original script	Soft skilled trained
32	Original script	Soft skilled trained
26	Original script	Soft skilled trained
37	Original script	Soft skilled trained
33	Original script	Soft skilled trained
30	Original script	Soft skilled trained
35	Original script	Soft skilled trained
30	Original script	Soft skilled trained
31	Original script	Soft skilled trained
35	Original script	Soft skilled trained
34	Original script	Soft skilled trained
26	Original script	Soft skilled trained
33	Original script	Soft skilled trained
34	Original script	Soft skilled trained
34	Original script	Soft skilled trained
33	Original Script	Soft skilled trained
33	Original Script	Soft skilled trained

Table B.1 (continued)

Income per sale	Script	Training
30	Original Script	Soft skilled trained
36	Original Script	Soft skilled trained
29	Original Script	Soft skilled trained
26	Original Script	Soft skilled trained
29	Original Script	Soft skilled trained
32	Original Script	Soft skilled trained
27	Original Script	Soft skilled trained
26	Original Script	Soft skilled trained
32	Original Script	Soft skilled trained
27	Original Script	Soft skilled trained
40	Original Script	Soft skilled trained
30	Original Script	Soft skilled trained
25	Original Script	Soft skilled trained
36	Original Script	Soft skilled trained
33	Original Script	Soft skilled trained
27	Original Script	Soft skilled trained
34	New script	Soft skilled trained
31	New script	Soft skilled trained
26	New script	Soft skilled trained
36	New script	Soft skilled trained
33	New script	Soft skilled trained
25	New script	Soft skilled trained
36	New script	Soft skilled trained
32	New script	Soft skilled trained
27	New script	Soft skilled trained
37	New script	Soft skilled trained
33	New script	Soft skilled trained
24	New script	Soft skilled trained
35	New script	Soft skilled trained
32	New script	Soft skilled trained
25	New script	Soft skilled trained
34	New script	Soft skilled trained
32	New script	Soft skilled trained
28,6	New script	Soft skilled trained
37	New script	Soft skilled trained
32	New script	Soft skilled trained
26	New script	Soft skilled trained
36	New script	Soft skilled trained
31	New script	Soft skilled trained
31	New script	Soft skilled trained
35,1	New script	Soft skilled trained
37	New script	Soft skilled trained
26,4	New script	Soft skilled trained
31,3	New script	Soft skilled trained

Table B.1 (continued)

Income per sale	Script	Training
36,3	New script	Soft skilled trained
29,3	New script	Soft skilled trained
33,4	New script	Soft skilled trained
36,2	New script	Soft skilled trained
26,2	New script	Soft skilled trained
35,4	New script	Soft skilled trained
35	New script	Soft skilled trained
30,6	New script	Soft skilled trained
39,2	New script	Soft skilled trained
28,8	New script	Soft skilled trained
26	New script	Soft skilled trained
36,3	New script	Soft skilled trained
27,4	Original script	Objection skilled trained
27,5	Original script	Objection skilled trained
30,8	Original script	Objection skilled trained
27,9	Original script	Objection skilled trained
26,6	Original script	Objection skilled trained
30	Original script	Objection skilled trained
27	Original script	Objection skilled trained
25,6	Original script	Objection skilled trained
27,2	Original script	Objection skilled trained
25,7	Original script	Objection skilled trained
23,3	Original script	Objection skilled trained
21,7	Original script	Objection skilled trained
24,1	Original script	Objection skilled trained
21,5	Original script	Objection skilled trained
25,2	Original script	Objection skilled trained
22,6	Original script	Objection skilled trained
22,4	Original script	Objection skilled trained
25,5	Original script	Objection skilled trained
25,8	Original script	Objection skilled trained
26,5	Original script	Objection skilled trained
26,5	Original script	Objection skilled trained
23	Original script	Objection skilled trained
26,2	Original script	Objection skilled trained
28,5	Original script	Objection skilled trained
25,9	Original script	Objection skilled trained
32,5	Original script	Objection skilled trained
28,8	Original script	Objection skilled trained
28,3	Original script	Objection skilled trained
28,5	Original script	Objection skilled trained
29,3	Original script	Objection skilled trained
30,5	Original script	Objection skilled trained
30,8	Original script	Objection skilled trained

Table B.1 (continued)

Income per sale	Script	Training
28,2	Original script	Objection skilled trained
29,1	Original script	Objection skilled trained
29,4	Original script	Objection skilled trained
29,9	Original script	Objection skilled trained
29,8	Original script	Objection skilled trained
30,2	Original script	Objection skilled trained
31,4	Original script	Objection skilled trained
32	Original script	Objection skilled trained
29	New script	Objection skilled trained
29,2	New script	Objection skilled trained
28,2	New script	Objection skilled trained
29,7	New script	Objection skilled trained
31	New script	Objection skilled trained
29	New script	Objection skilled trained
26	New script	Objection skilled trained
29	New script	Objection skilled trained
32	New script	Objection skilled trained
24	New script	Objection skilled trained
33	New script	Objection skilled trained
31	New script	Objection skilled trained
30	New script	Objection skilled trained
30	New script	Objection skilled trained
28	New script	Objection skilled trained
25	New script	Objection skilled trained
33	New script	Objection skilled trained
30	New script	Objection skilled trained
32	New script	Objection skilled trained
33	New script	Objection skilled trained
31	New script	Objection skilled trained
26	New script	Objection skilled trained
29	New script	Objection skilled trained
29,3	New script	Objection skilled trained
25	New script	Objection skilled trained
32	New script	Objection skilled trained
32	New script	Objection skilled trained
24	New script	Objection skilled trained
35	New script	Objection skilled trained
30	New script	Objection skilled trained
33	New script	Objection skilled trained
35	New script	Objection skilled trained
30	New script	Objection skilled trained
33	New script	Objection skilled trained
34	New script	Objection skilled trained
28	New script	Objection skilled trained

Table B.1 (continued)

Income per sale	Script	Training
23	New script	Objection skilled trained
36	New script	Objection skilled trained
33	New script	Objection skilled trained
24	New script	Objection skilled trained

B.6 Verification of ANOVA assumptions

B.6.1 General

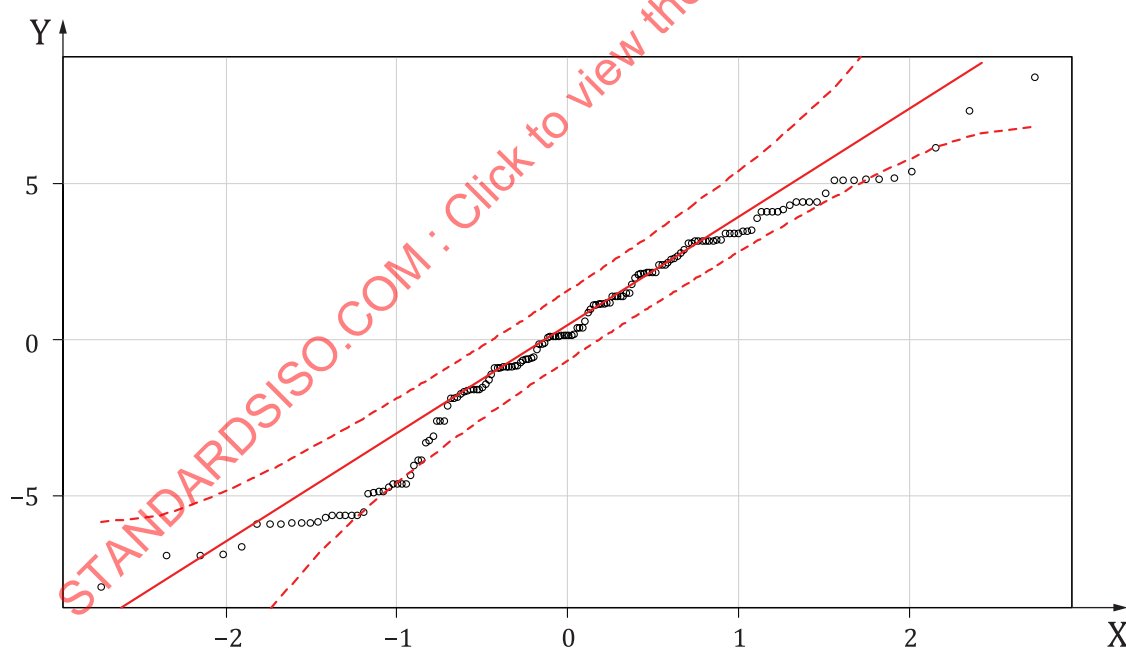
Verification of ANOVA assumptions mainly include the aspects of normality, independence and homogeneity.

B.6.2 Test of independence

Independence can be checked by investigating the method of data collection. The data are collected randomly.

B.6.3 Test of normality

The normality test needs to confirm that all groups follow a normal distribution. Q-Q plot is used to check the sample's normality by the help of R software²⁾ and the package "car" is necessary. The Q-Q plot in Figure B.1 shows that normal assumption is nearly satisfied.



Key

X norm quantiles
Y residuals

Figure B.1 — Normal Q-Q plot

2) R software is an example of a suitable product available commercially. This information is given for the convenience of users of this document and does not constitute an endorsement by ISO of this product.

B.6.4 Test for homogeneity of variance

Next, one tests the homogeneity, and R is applied to do equivalence of variance tests, including Bartlett test and Levene's test. The results are displayed in [Table B.2](#). The p -value of Bartlett's test is 0,141 3, which is greater than 0,05. In addition, another Levene's test displays that the p -value is 0,082 63, which is greater than 0,05 as well. Both above illustrations conclude that the equivalence of variance is satisfied.

Table B.2 — Result of homogeneity test

Methods	DF	Statistic value	p -value
Bartlett test	3	K-squared: 5,456 7	0,141 3
Levene's test	3	F value: 2,269 6	0,082 63

NOTE Levene's test is an inferential statistic used to assess the equality of variances for a variable calculated for two or more groups.

B.7 Undertaking ANOVA analysis

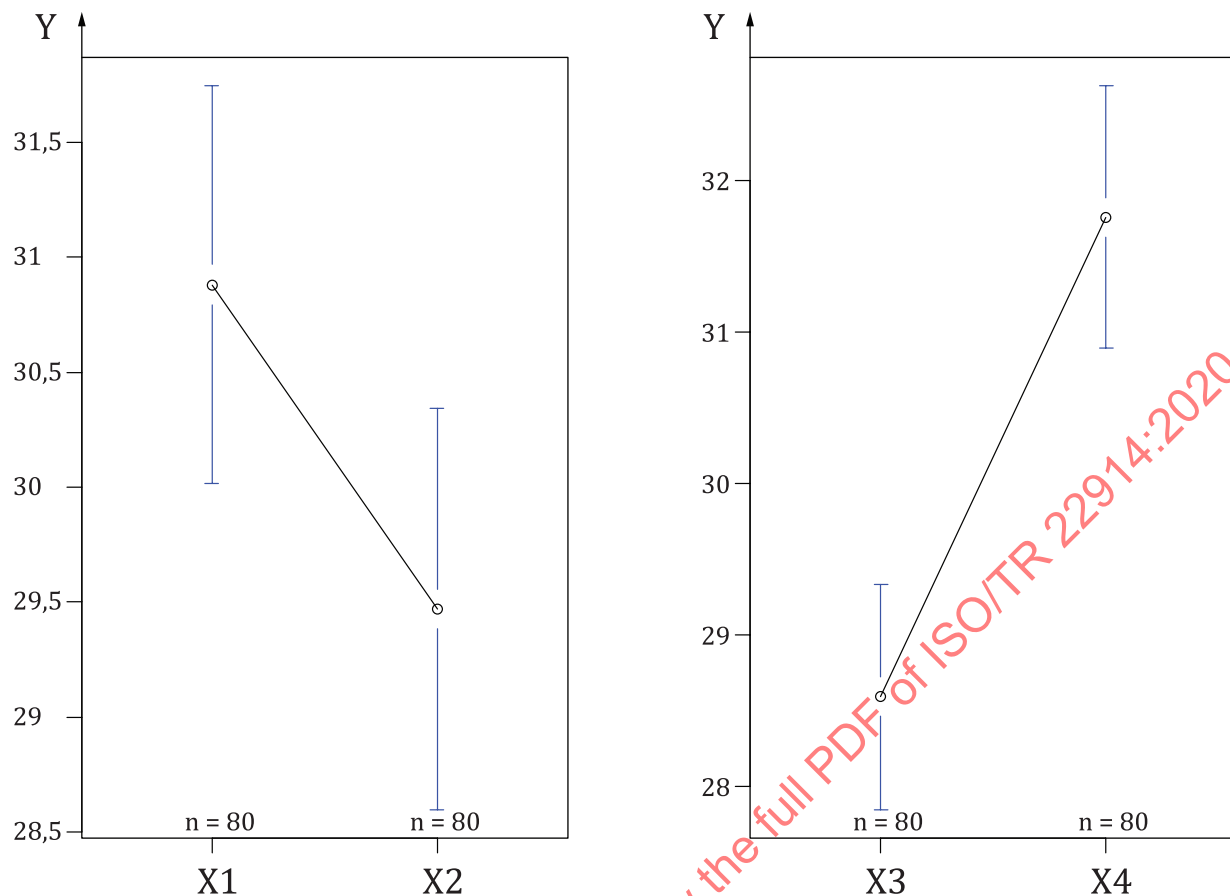
First the hypotheses H_0 and H_1 are stated as follows.

H_0 : there is no difference in income per sale among different scripts and training modules.

H_1 : there is a difference in income per sale between at least one script and/or training module and the others.

Graphical analysis

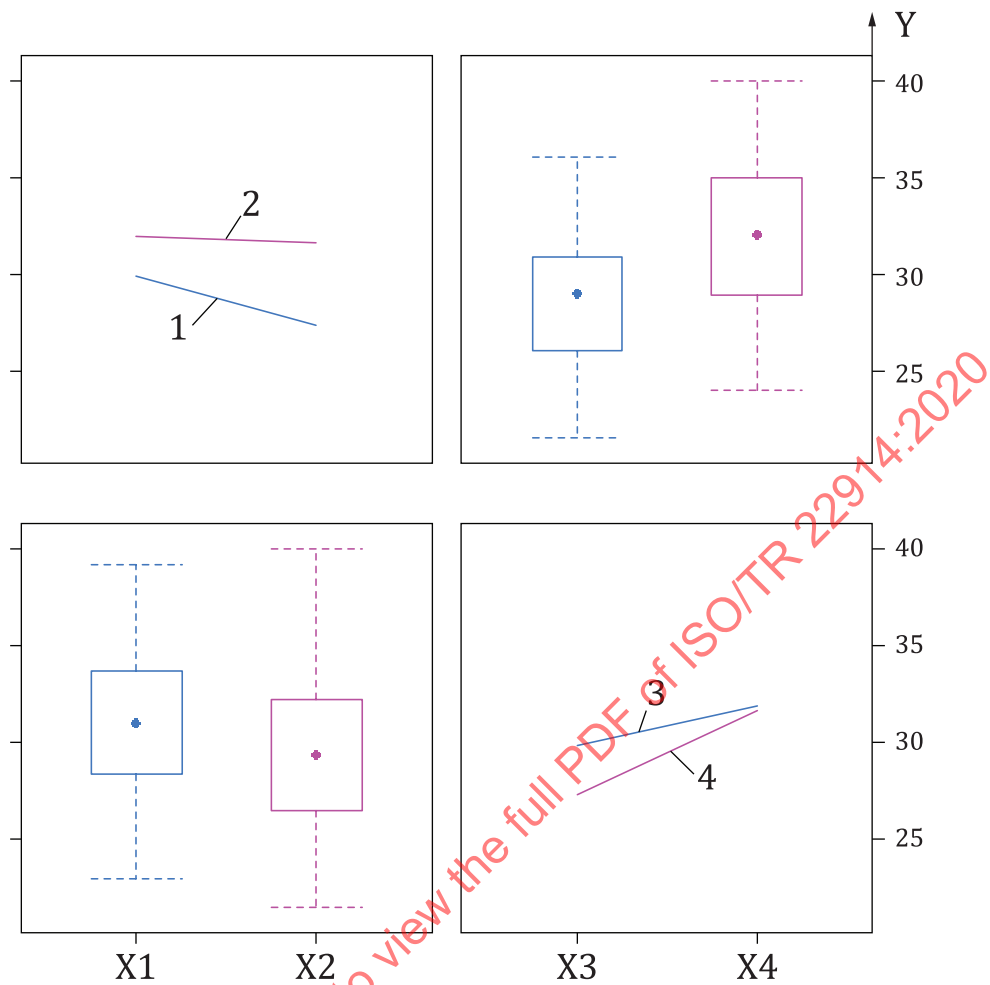
[Figure B.2](#) shows the main effects plots for the factors script and training. First, the main effects plots are considered. If the lines on the graphs are not horizontal, one can say that the main effect is present. This analysis suggests that using the new script increases the income per sale. Also, the analysis suggests that those who have been trained in soft skills have a higher income per sale than those who were trained in objection handling.

**Key**

- X1 new script
- X2 original script
- X3 objection handling
- X4 soft skills
- Y income

Figure B.2 — Main effects mean plot with 95 % confidence intervals

Figure B.3 shows the two factors' interaction effects and box plots respectively. If the lines on the graphs are not parallel, one can say that the interaction effect is present. As the lines on the graph are not parallel, the interaction effect exists.



Key	
X1 new script	1 trained, objection handling
X2 original script	2 trained, soft skills
X3 objection handling	3 script, new
X4 soft skills	4 script, original
Y income	

Figure B.3 — Interaction plot and box plots of data

ANOVA procedure

There is a need to review the p -value of the ANOVA tests to confirm whether there has been an impact in each factor. The result is in [Table B.3](#). All of the p -values are less than 0,05 and therefore it can be concluded that both two factors and interaction terms have significant effects on the output (i.e. the income per sale) at the 0,05 α -level.

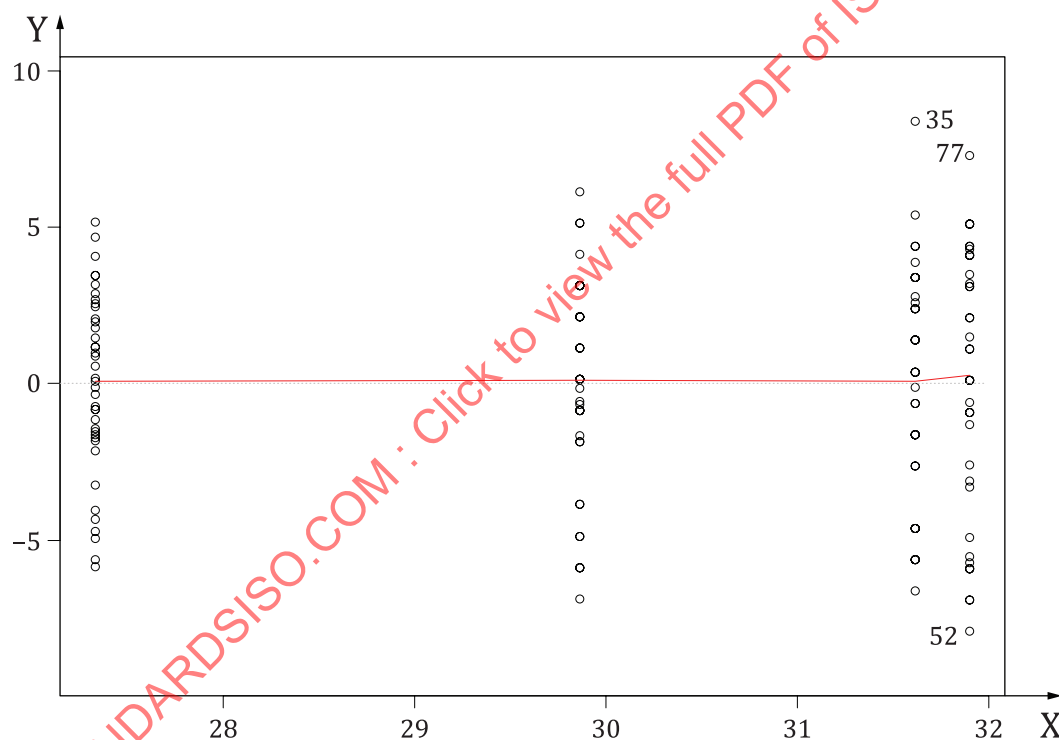
Table B.3 — Analysis of variance results

Income per sale versus script, training					
Source	DF	SS	MS	F-value	p-value
Script	1	79,5	79,5	6,365	0,012 6
Training	1	400,7	400,7	32,069	0,000
Script × training	1	50,4	50,4	4,034	0,046 3
Error	156	1 949,2	12,5		

NOTE Although the p -value for the interaction term is below 0,05, it is the experimenter's judgement to assess whether it is significant or not.

Residual analysis

Figure B.4 shows the residual vs fitted values plot. The figure shows the residuals are randomly scattered about zero and have approximately the same scatter for all fitted values. The fairly evenly distributed errors show that variance equality is satisfied within all groups.



Key

X fitted values
Y residuals

Figure B.4 — Residual vs fitted values plot

B.8 Further analysis

Multiple comparisons

The p -values in Table B.3 indicate at least one significant difference between the factor level means, but do not identify the means that differ. To determine which mean differences are significant, one can test them using the Tukey comparison method.

Table B.4 — Differences in mean levels

Group	Script:Training	diff	lwr	upr	p-value
1	Original:Objection - New:Objection	-2,532 5	-4,585 1	-0,479 9	0,008 8
2	New:Soft - New:Objection	2,042 5	-0,010 1	4,095 1	0,051 7
3	Original:Soft - New:Objection	1,755 0	-0,297 6	3,807 6	0,122 2
4	New:Soft - Original:Objection	4,575 0	2,522 4	6,627 6	0,000 0
5	Original:Soft - Original:Objection	4,287 5	2,234 9	6,340 1	0,000 0
6	Original:Soft - New:Soft	-0,287 5	-2,340 1	1,765 1	0,983 5
NOTE Diff indicates differences between means of the two levels. Lwr and Upr show the lower and upper confidence interval at 95 %.					

The *p*-value presents adjustment for the multiple comparisons.

[Table B.4](#) shows all groups' differences in mean levels. From the *p*-values, one can find that there are three groups which have significant difference. These groups are group 1, group 4 and group 5.

B.9 Conclusion

The obtained results show that the interaction between soft skill training and new script is synergistic (i.e., they reinforce each other for increasing the income per sale). The sales department has therefore decided to deploy jointly the new script and soft skills training for all sales agents.

Annex C (informative)

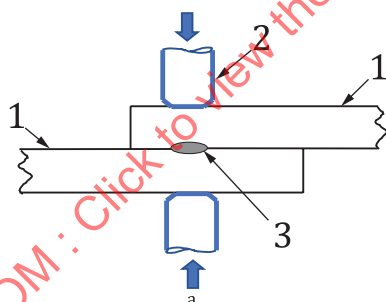
Strength of welded joint

C.1 Stating objectives

A process development team wants to optimize parameters of a spot-welding process. During the improve phase of a Six Sigma project, they want to maximize the strength of welded joints. The team uses their experience and technical knowledge to decide approximate settings of the two parameters which are current and force. A sketch of the process is shown in [Figure C.1](#). The team decides to conduct an experiment with the factors and settings (usually called levels) as shown in [Table C.1](#).

Table C.1 — Factors and settings

Factor	A: Current kA	B: Force kN
Low level	300	10
High level	450	20



Key

- 1 steel plate
- 2 electrodes
- 3 weld nugget
- a Force.

Figure C.1 — Sketch of the process

C.2 Data collection plan

In statistically designed experiments, it is essential that the data be collected by conducting experimental trials randomly in order to minimize the effect of factors which are not under control of the experimenter. Such uncontrolled factors are called noise factors. A single replicate of the design has L^F or $2^2 = 4$ trials (also called runs).

C.3 Variables description

The considered response variable is the strength of welded joints. (The maximum tension that the weld can bear.)

There are two factors in the experiment:

- factor A: force, which has two levels: low level and high level;
- factor B: current, which also has two levels: low level and high level.

In addition to this, it is necessary to analyse the interaction $A \times B$, that is force $\times$ current.

C.4 Measurement system considerations

The experimenters ensured that the measurement systems used for force, current and strength were adequate for the purpose. Strength of the joint is a critical characteristic. The measurement system for strength was qualified by the laboratory audit team. All runs have the same set up process. Overall, the variability of a measurement system is small relative to the variability of the monitored process.

C.5 Performing data collection

When each factor level is performed at each level of the other factor(s), the experiment is called full factorial design. The number of trials (or runs) in a 'single replicate' full factorial design can be calculated as (levels) factors or L^F . Thus, in this case, the number of runs in a single replicate is $2^2 = 4$. A single replicate provides (4-1), that is three degrees of freedom. The degrees of freedom in ANOVA would be 1 for current, 1 for force, and 1 for the interaction current $\times$ force. But calculation of the F-ratio in ANOVA requires calculation of mean square (MS) values for each term, which equals (sum of squares)/(degrees of freedom) for the respective term. As there is not any degrees of freedom for error term, F-ratio for the ANOVA terms are indeterminate. Thus, the team realized that it would be necessary to replicate the design at least once. They decided to run three replicates of the design which means that the number of runs would be $4 \times 3 = 12$.

The data was collected by conducting the trials as the sequence in run order columns. [Table C.2](#) shows the design matrix along with the data of strength collected in the 12 trials.

Table C.2 — Raw data

Standard order	Run order	Current kA	Force kN	Strength kg
1	1	300	10	154
2	4	450	10	209
3	2	300	20	245
4	3	450	20	317
5	6	300	10	178
6	8	450	10	192
7	5	300	20	238
8	7	450	20	334
9	11	300	10	164
10	10	450	10	196
11	9	300	20	231
12	12	450	20	341

Power analysis

The experiment was conducted as per the column run order, while the column 'standard order' shows the order as per the design matrix. Power calculations are performed (see [Table C.3](#) for JMP³⁾ output) showing that 3 replicates are appropriate for taking into account both the main effect and the interactions. The output shows power of 0,99 or more for each of the three effects, i.e. the two main effects and the interaction effect, which means that if the effect is real, then there is 99 % chance of detecting it in the ANOVA analysis at alpha risk of 0,05, that is 5 %.

Table C.3 — Power analysis

Sources	alpha α	sigma σ	delta δ	number n	power
Current	0,050 0	10,315 845 417	31,58	12	1,000 0
Force	0,050 0	10,315 845 417	51,08	12	1,000 0
Current × force	0,050 0	10,315 845 417	14,75	12	0,990 4
Alpha (α) represents the significance level between 0 and 1 and, in this calculation, it automatically has a value of 0,05.					
Sigma (σ) represents the standard error of the residual error in the model. It is the square root of the mean square error.					
Delta (δ) represents the raw effect size. It is the square root of the sum of squares for the hypothesis divided by number (n).					

C.6 Verification of ANOVA assumptions

C.6.1 General

Verification of ANOVA assumptions mainly include the aspects of equality of variances within groups, normality and independence.

C.6.2 Test of independence

Independence is assured by the method of data collection.

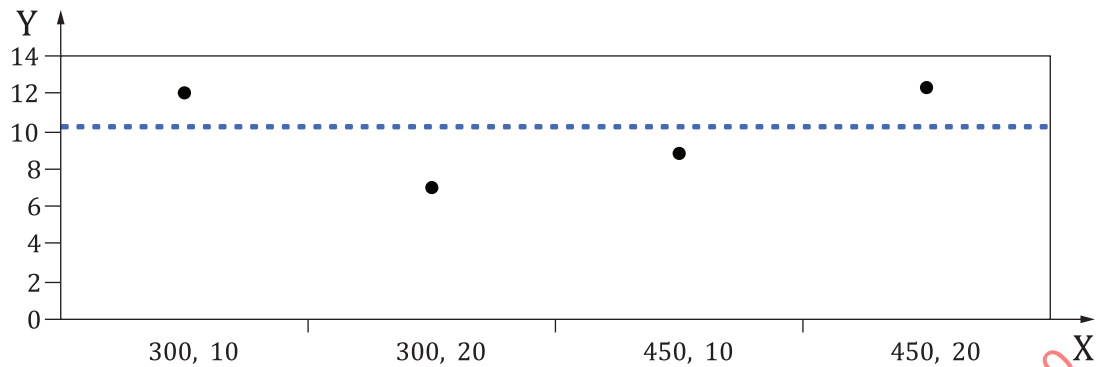
C.6.3 Test of normality

The normality of data is validated using the residuals in the ANOVA procedure (see [Figure C.5](#)).

C.6.4 Test for homogeneity of variance

To check equality of variances, tests of unequal variances are performed (see [Figure C.2](#)) in JMP software. As all the tests show that the value of Prob>F is greater than 0,05, it can be concluded that that variances within groups can be considered as equal.

3) JMP is an example of a suitable product available commercially. This information is given for the convenience of users of this document and does not constitute an endorsement by ISO of this product.

**Key**

X current (kA), force (kN)

Y standard deviation

Figure C.2 — Standard deviations of the four groups**Table C.4 — Numerical summaries of four groups**

Level (current, force)	Count	Std Dev	MeanAbsDif to Mean	MeanAbsDif to Median
300, 10	3	12,055 43	8,444 444	11,333 33
300, 20	3	7	4,666 667	7
450, 10	3	8,888 19	6,666 667	7
450, 20	3	12,342 34	9,111 111	10,333 33

Table C.5 — Four different tests for homogeneity of variance

Test	F Ratio	DFNum	DFDen	Prob > F
O'Brien	0,343 3	3	8	0,795
Brown-Forsythe	0,925 5	3	8	0,471 4
Levene	0,501 9	3	8	0,691 5
Bartlett	0,222 4	3		0,880 9

C.7 Undertaking ANOVA analysis

First, the hypotheses H_0 and H_1 are stated as follows:

H_0 : there is no difference in strength among different settings of current and force;

H_1 : there is a difference in strength among different settings of current and force.

Graphical analysis is performed. In experimental designs, it is customary to review main effect plots and interaction plots.

Graphical analysis

[Figure C.3](#) and [Figure C.4](#) show the main effects plot and the interaction plot for the factors current and force and for the interaction current $\times$ force in JMP software separately. It is clear that increasing current as well as force increases the strength. However, the effect of force is somewhat stronger compared to the effect of current, as indicated by the steepness of lines. The interaction plot shows that the effect of current is greater when the force is set at 20 than if it is set at 10. This phenomenon is known as interactions between the factors. Parallel lines in the interaction plot indicate a lack of interaction

between the factors, while the extent of non-parallelism indicates the extent of the interaction effect. It can be expected in ANOVA that the main effects and the interaction are likely to be statistically significant.

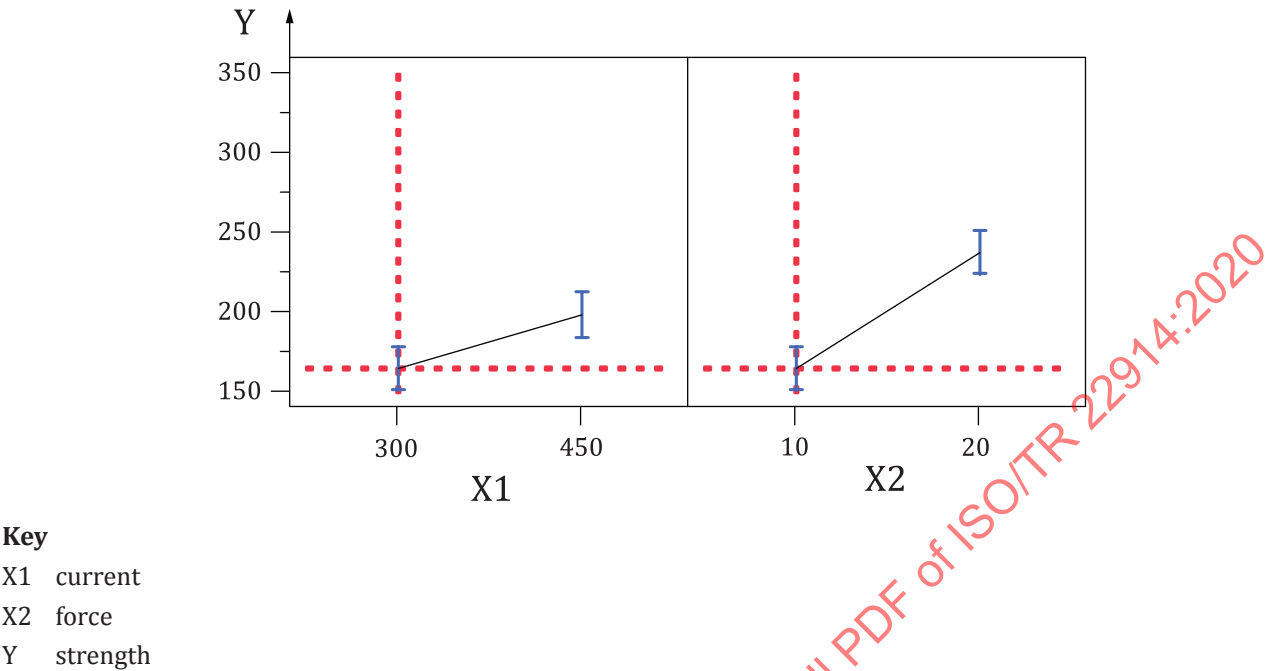


Figure C.3 — Main effects

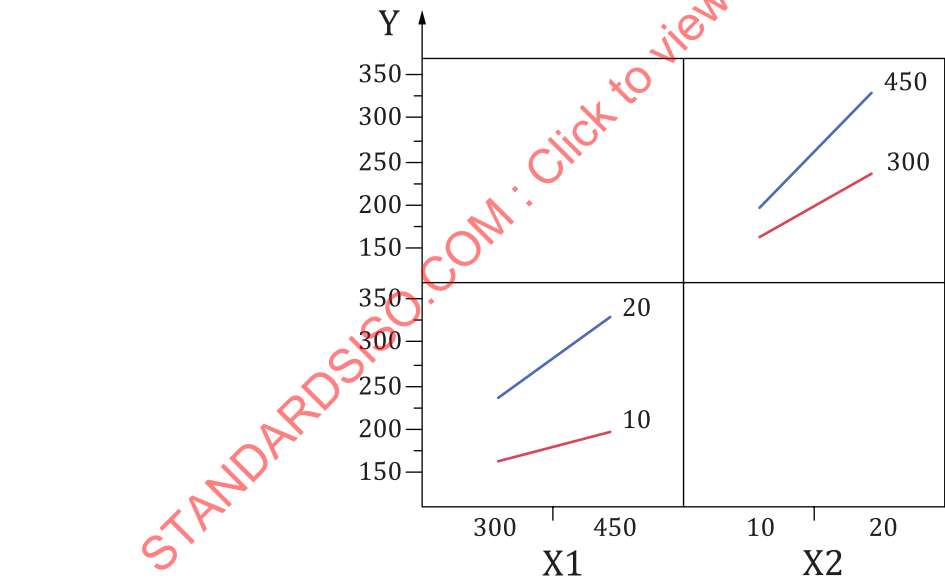


Figure C.4 — Interaction effects

ANOVA procedure

ANOVA is performed with JMP software. The ANOVA table output is shown in [Table C.6](#). It is observed that the degrees of freedom for the model are 3, while the degrees of freedom for error are 8. The model summary shows an R-square value of 0,9818, indicating that 98,18 % of variation is explained by the two factors in the experiment. The three degrees of freedom are further split into current, force and interaction as 1 each. As the Prob>F (or the p -value) of each term is <0,01, the effects of current, force and the interaction current $\times$ force are statistically significant at 99 % confidence level.

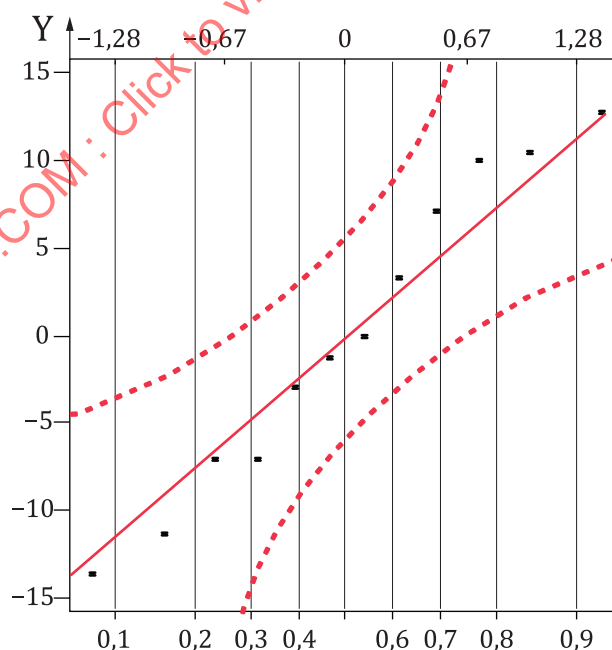
Table C.6 — Analysis of variance for strength

Source	DF	Adj SS	Adj MS	F-value	p -value
Current	1	11 970,1	11 970,1	112,48	0,000
Force	1	31 314,1	31 314,1	294,26	0,000
Current*Force	1	2 610,8	2 610,8	24,53	0,001
Error	8	851,3	106,4		
R-square: 98,18 %		R-square (adjusted):97,50 %			

Residual analysis

Residual plots shown in [Figure C.5](#), [C.6](#) and [C.7](#) support validity of ANOVA as follows.

- Normal probability plot (see [Figure C.5](#)) – The outer dotted lines on the plot are 95 % confidence intervals for the individual percentiles. Since all the points fall within the corresponding 95 % confidence intervals and the points on the normal probability plot roughly follow a straight line, it can be assumed that the residuals do not deviate substantially from a normal distribution. In other words, residual errors are distributed normally as seen in the normal probability plot.

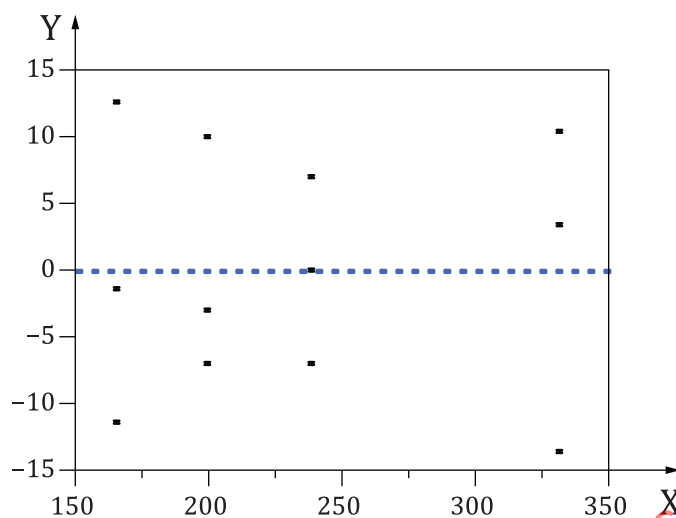


Key

Y residuals

Figure C.5 — Normal probability plot of residuals

- ii. Residual vs fits graph (see [Figure C.6](#)) – It shows the residuals are randomly scattered about zero and have approximately the same scatter for all fitted values. Therefore, the constant variance assumption does not appear to be violated.

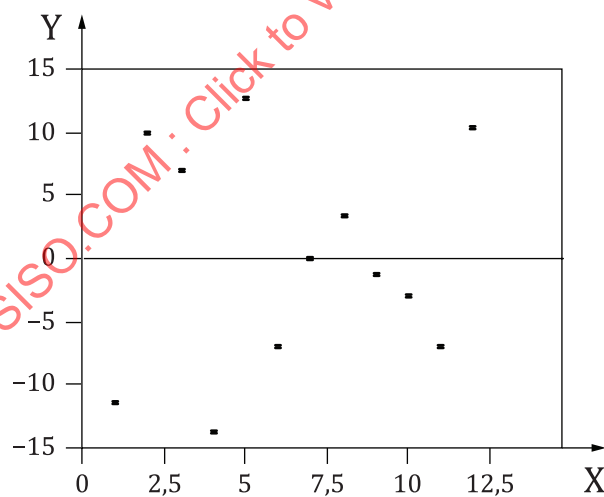


Key

X fitted value
Y residuals

Figure C.6 — Residual vs strength predicted plots

- iii. The plot of the residuals by row number is shown in [Figure C.7](#). It does not show any pattern. Therefore, there is no apparent time dependence in the residuals.



Key

X observation order
Y residuals

Figure C.7 — Residual plot by row

C.8 Further analysis

Multiple comparisons

The p -values in the ANOVA table (Table C.6) indicate differences between the factor level means, but do not identify the means that differ. To determine which mean differences are significant, one can test them by using Tukey's method, which compares all possible pairs of level means of the specified factors and interaction terms. The results are in Table C.7.

Table C.7 — Tukey HSD pairwise comparison

Current kA	Force kN	Current kA	Force kN	Difference	Standard error	DF	t ratio	Prob > t	95 % lower limit	95 % upper limit
300	10	300	20	-72,667	8,422 853	8	-8,63	0,000 1	-99,64	-45,694
300	10	450	10	-33,667	8,422 853	8	-4	0,016 7	-60,64	-6,694
300	10	450	20	-165,333	8,422 853	8	-19,63	<,000 1	-192,306	-138,36
300	20	450	10	39	8,422 853	8	4,63	0,007 3	12,027	65,973
300	20	450	20	-92,667	8,422 853	8	-11	<,000 1	-119,64	-65,694
450	10	450	20	-131,667	8,422 853	8	-15,63	<,000 1	-158,64	-104,694

Table C.7 shows that all the pairs of factor levels are significantly different based on the multiple comparison tests.

Further analysis can be performed by calculation of main effects and interactions. The derived regression equation, establishing contour and surface plots, can be used for response optimization to find settings for a target response.

C.9 Conclusion

The design of experiment and ANOVA during the improve phase provided the Six Sigma project team insights on how to optimize the process parameters to maximize the strength of welded joint. For this, the team performed calculations of the main effects and the interaction effect, and calculated the optimum settings by creating a regression equation of the process.

The effects of current, force and interaction were statistically significant at 95 % confidence level. These two factors contributed about 98 % to the total variation in strength. Furthermore, it can be found that the optimized parameters of spot-welding process to maximize strength of welded joints are 450 kA for current and 20 kN for force.

Annex D (informative)

Water consumption in a petroleum enterprise

D.1 Stating objectives

A petroleum enterprise not only produces energy, but also consumes energy. Energy consumption accounts for quite a high proportion in enterprise cost. Energy consumption cost is the controllable part of the enterprise cost, so to reduce the energy consumption of the unit product is an important factor of controlling production cost whether effectively or not. Energy conservation is beneficial to reducing the production cost, optimizing the cost structure of the enterprise, thus improving enterprise's competition ability in the market economy. The particularity of petroleum production determines that the petroleum industry is highly dependent on water resources, and the amount of water used are relatively large. Beijing Sunlight science and technology consulting company is asked to deploy the Six Sigma project by one petroleum enterprise. This project intends to minimize the water consumption. First, the technical manager hopes to decrease the water consumption caused by persons through finding out the significant factors. The manager learned that the petroleum enterprise has a system of three shifts by five teams. The practice typically sees the day divided into shifts, set periods of time during which different teams of workers perform their duties. The administrators want to know whether there are significant differences in water consumption among the five teams and three time shifts through experiments.

The experimenter has assured that there are not any uncontrolled variables in the environment been introduced in the experiment sequence. This is a general methodological requirement that needs to be verified prior to any analysis. This experiment has been conducted during the analyse phase of DMAIC (define, measure, analyse, improve, control).

D.2 Data collection plan

In the experiments, the water consumption of every team was recorded in their every continuous working time in 30 days. The continuous working time of each team is eight hours. Every day is divided into three parts (0:00-8:00; 8:00-16:00; 16:00-24:00) which correspond to three time shifts. The night shift is from 0:00-8:00. The morning shift is from 8:00-16:00. The middle shift is from 16:00-24:00.

D.3 Variables description

The considered response variable is the water consumption of different teams.

There are two factors:

- factor A: team, which have five levels;
- factor B: time shifts, which have three levels: morning shift, middle shift and night shift.

D.4 Measurement system considerations

To ensure the reliability of measurement, a standard operating procedure (SOP) has been provided to each team, for consistently collecting and verifying the data. The water consumption amount is recorded by the same water-meter readers. The SOP provides instructions, in particular on how to ensure that readings and scales are clearly visible and how the key elements within the water-meter work under the same conditions.

The sources of possible variations for this measurement system are controlled. The capability of a measurement system is acceptable for a given monitored process. (In the measurement process, generally one water-meter is used.)

D.5 Performing data collection

[Table D.1](#) provides the raw data of water consumption (the unit is cubic numbers).

Table D.1 — Raw data

Shift	Team one	Team two	Team three	Team four	Team five
morning shift	41 146	41 664	41 605	41 627	41 258
	41 342	41 392	41 568	41 273	41 237
	41 096	41 148	41 382	41 392	41 467
	41 684	41 762	40 837	41 285	41 433
	41 378	41 570	41 526	40 965	41 131
	41 099	41 287	41 347	41 198	41 127
	41 416	41 294	41 321	41 203	41 386
	41 477	41 526	40 974	41 284	41 484
	41 437	41 497	41 236	41 564	41 271
	41 224	41 442	41 472	41 483	41 511
middle shift	41 197	41 761	41 520	41 721	41 319
	41 410	41 465	41 487	41 335	41 295
	41 141	41 198	41 316	41 464	41 546
	41 783	41 777	40 817	41 348	41 509
	41 449	41 659	41 448	40 998	41 180
	41 145	41 350	41 284	41 252	41 175
	41 491	41 358	41 260	41 258	41 458
	41 557	41 611	40 942	41 346	41 565
	41 514	41 579	41 182	41 652	41 332
	41 281	41 519	41 399	41 563	41 595
night shift	41 836	42 388	42 235	42 349	41 956
	42 045	42 099	42 200	41 971	41 933
	41 782	41 838	42 017	42 098	42 178
	42 410	41 927	41 485	41 984	42 142
	42 083	42 288	42 158	41 643	41 820
	41 786	41 986	41 984	41 891	41 815
	42 124	41 994	41 958	41 896	42 092
	42 188	42 242	41 619	41 983	42 196
	42 147	42 210	41 875	42 282	41 969
	41 919	42 151	42 106	42 195	42 225

D.6 Verification of ANOVA assumptions

D.6.1 Test of independence

Independence can be checked by investigating data collection methods. After a sample survey, the data collection method ensures independence.

D.6.2 Test of normality

The normality of data is validated using the normal probability plot of residuals in a residual analysis procedure (see [Figure D.4](#)).

D.6.3 Test for homogeneity of variance

One can use the modified Levene's test to test the homogeneity of variance, and the results are in [Table D.2](#). From [Table D.2](#) it can be concluded that the variances of all cells are equal.

Table D.2 — Results of modified Levene's test

H_0 : the variances of all cells are equal		
H_1 : the variances of all cells are not equal		
Test level	Critical values	
	Upper	Lower
$\alpha = 5 \%$		1,77
$\alpha = 1 \%$		2,22
$\alpha = 0,1 \%$		2,82
Test results		
Null hypothesis is not rejected		

NOTE The modified Levene's Test is used to determine if the variances from multiple treatments (or processes) are the same.

D.7 Undertaking ANOVA analysis

First the hypotheses H_0 and H_1 are stated:

H_0 : there is no difference in water consumption among different teams and shifts;

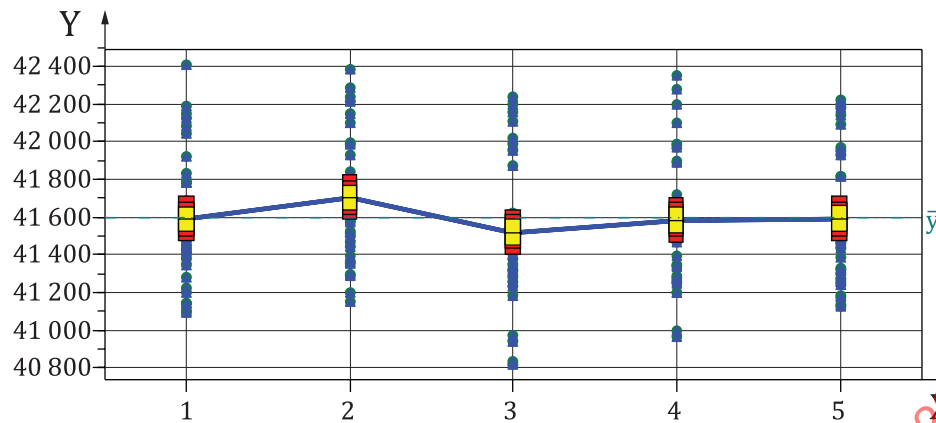
H_1 : there is a difference in water consumption between at least one subgroup team and/or shift and the others.

Then, a graphical analysis is performed.

Graphical analysis

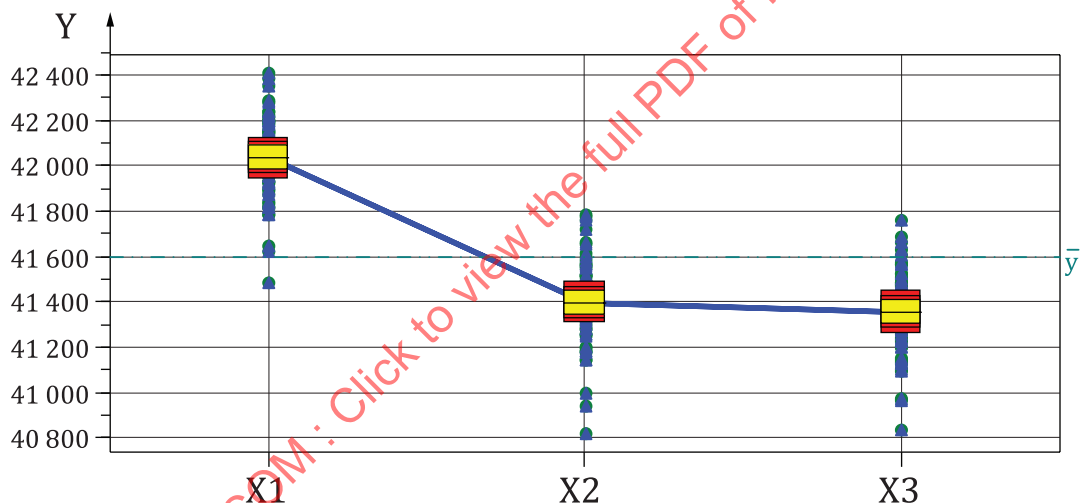
The main effect of the data is displayed for team and time shift to assess the difference of means of different team and different time shift. [Figure D.1](#) and [Figure D.2](#) show the main effects plot for the factors team and time shift, respectively. [Figure D.3](#) shows the interaction plot for team $\times$ shift in Q-DAS⁴⁾ software.

4) Q-DAS is an example of a suitable product available commercially. This information is given for the convenience of users of this document and does not constitute an endorsement by ISO of this product.

**Key**

X team

Y water consumption

Figure D.1 — Main effect plot for team**Key**

X1 night shift

X2 middle shift

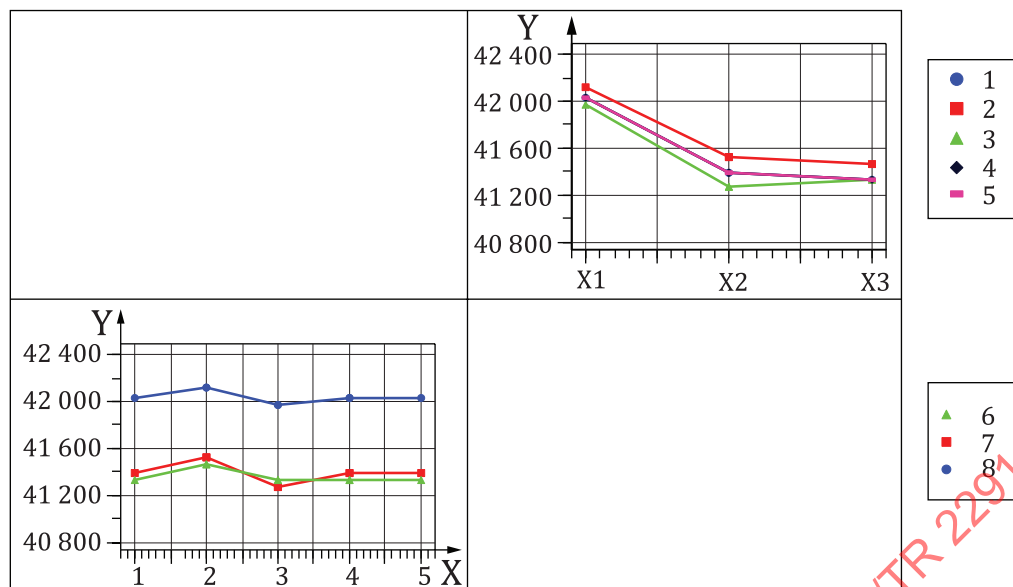
X3 morning shift

Y water consumption

Figure D.2 — Main effect plot for time shift

The main effect figure shows that the water consumption of the five teams is not much different, and the water consumption of the three-time shift is significantly different. The water consumption in the night shift (0:00-8:00) has a much larger value than the morning shift and the middle shift.

Parallel lines in the interaction plot indicate a lack of interaction between the factors, while the extent of non-parallelism indicates the extent of the interaction effect. From [Figure D.3](#), it can be expected in ANOVA that the interaction effects are likely to be statistically insignificant.

**Key**

X	team	3	team 3
Y	water consumption	4	team 4
X1	night shift	5	team 5
X2	middle shift	6	morning shift
X3	morning shift	7	middle shift
1	team 1	8	night shift
2	team 2		

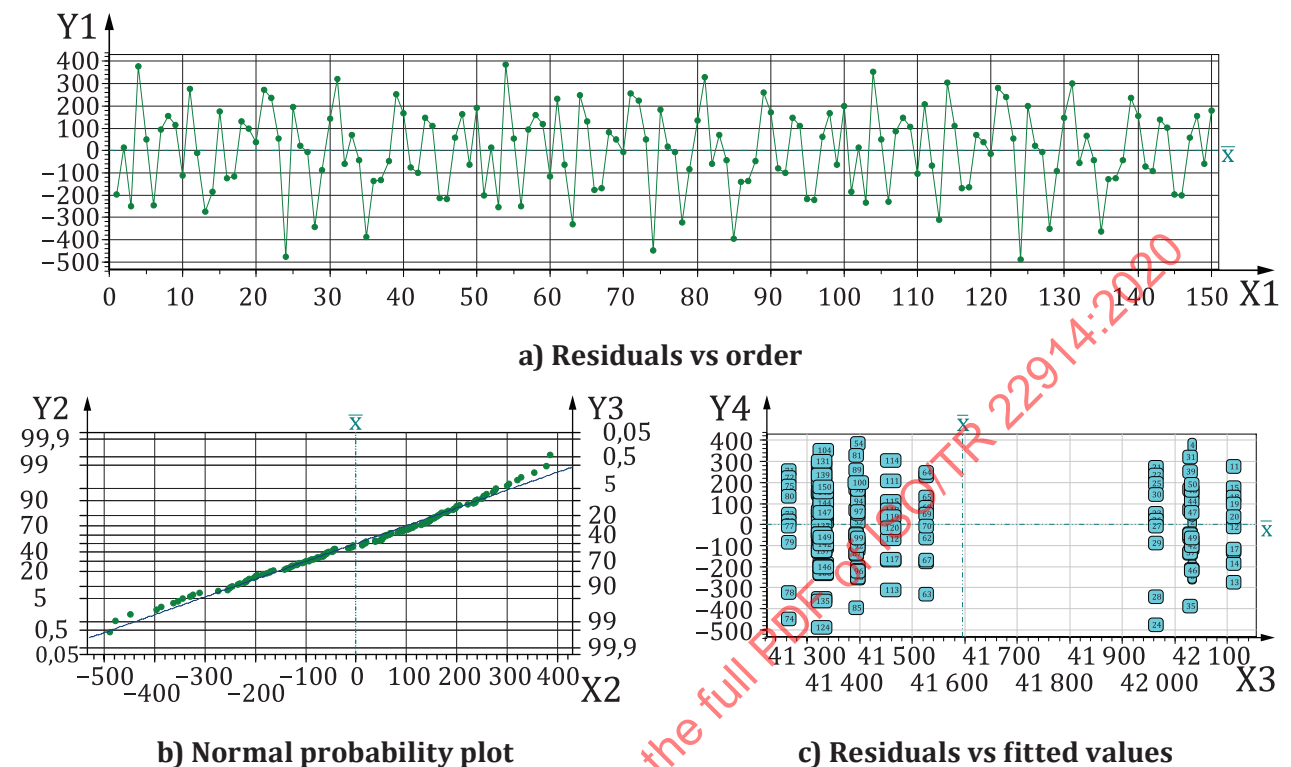
Figure D.3 — Interaction plot for team and time shift**ANOVA procedure**

There is a need to review the p -value of the ANOVA tests to confirm whether there has been an impact in each factor. The analysis results are in [Table D.3](#). The p -value for team is 0,015 which is greater than 0,01. So one can conclude that there are no significant differences in water consumption among the five teams at the 0,01 α -level. The p -value for shift is 0,000, which is smaller than 0,01, which means the effect of time shift on the water consumption is significant at the 0,01 α -level. The p -value for team $\times$ shift is 0,980, which is greater than 0,01, which means no interaction effect exists between team and time shift.

Table D.3 — Results of ANOVA: water consumption vs team and shift

Source	DF	Sum square	Mean square	F value	p -value
team	4	510 103	127 526	3,22*	0,015
shift	2	14 500 537	7 250 269	183,29**	0
Team $\times$ shift	8	79 110	9 889	0,250	0,980
Error	135	5 340 212	39 557		
R-square: 73,861 %		R-square (adjusted): 71,15 %			

Residual analysis



Key

- X1 observation order
- X2 residuals
- X3 fitted value
- Y1 residuals
- Y2 percent (p[%])
- Y3 percent (1-p [%])
- Y4 residuals

Figure D.4 — Residual plots

The residual plots are used to check the assumptions about the error distribution. [Figure D.4](#) is the residual plot.

- **Residuals versus order** – The plot of the residuals versus order does not show any pattern. Therefore, there is no time dependence in the residuals.
- **Normal probability plot** – Because the points on the normal probability plot roughly follow a straight line, one can assume that the residuals do not deviate substantially from a normal distribution.
- **Residuals versus fitted values** – The constant variance assumption does not appear to be violated because the residuals are randomly scattered about zero and have approximately the same scatter for all fitted values.

D.8 Further analysis

Multiple comparisons

From the ANOVA table ([Table D.3](#)) one can see that there is significant difference among water consumption of three times shift. To determine which mean differences are significant, one can test them using one of the comparison methods.

Pairwise-Bonferroni method compares all possible pairs of level means of the time shift. The results are in [Table D.4](#).

Table D.4 — Multiple comparison pairwise-Bonferroni

x_i	$i-j$	μ_{i-j}	s_{i-j}	t_{i-j}	$P_{i-j} > t $
x_1 = morning shift	2-3	-637,7	39,78	-16,03***	<0,000 1
x_2 =middle shift	1-3	-679,4	39,78	-17,08***	<0,000 1
x_3 =night shift	1-2	-41,66	39,78	-1,047	0,890
μ_{i-j} denotes the mean difference between i and j . s_{i-j} denotes the total standard deviation. t_{i-j} denotes the observed value of t-statistics for testing the difference of two groups.					

The results of [Table D.4](#) indicate:

- the morning shift and middle shift are not statistically different from one another;
- the night shift is statistically different from the morning shift as well as middle shift.

D.9 Conclusion

The project objective is to find the significant factors that are related to water consumption. Based on the ANOVA analysis of the water consumption, it can be concluded that:

- the team is not a significant factor that affects water consumption;
- the time shift is a significant factor that affects water consumption;
- the water consumption in night shift for all teams is largest.

Because water consumption in the night shift for all teams is largest, the director concluded that, as it is tiring to work in the night shift, some workers forget to turn off the faucet timely. Then the director decided to install touchless faucets in some places. After that, data is collected again to validate the effect of the improvements.

Annex E (informative)

The hub total hours used on a task

E.1 Stating objectives

A project leader wanted to investigate the time taken to process customer orders through a system. One technique used was to ask the team for comments on the process. They commented that the performance seemed to be different on different days of the week. Their mentor asked for the data to verify whether these perceived differences were statistically different. So the project intended to understand whether the hours used on a task varied significantly by day of the week (one of the ideas suggested by the team).

This experiment has been conducted during the define phase of the DMAIC-T (define, measure, analyse, improve, control, transfer), trying to clearly define the situation (excess hours booked to the task in the hub above what was budgeted).

E.2 Data collection plan

The data required is all automatically collected and stored in company databases. The collection plan is therefore to select which time-period will be used for the analysis and which database fields will be used.

E.3 Variables description

The considered response variable is the hub total hours (total time in hours, spent by all employees on one specific logistics activity).

There is one factor: day of the week, which have six levels: Friday, Sunday, Monday, Tuesday, Wednesday, and Thursday.

E.4 Measurement system considerations

When using existing data from company databases, it is always prudent to scrutinise how that data has been collected (its origins and collection process) and what cleansing has been performed prior to storage. This scrutiny also entails a preliminary data-visualisation exercise (in both the time domain and the frequency domain) to identify and investigate any data points which appear unusual.

E.5 Performing data collection

[Table E.1](#) provides the raw data of hub total hours on days of the week.

Table E.1 — Raw data

Friday	Sunday	Monday	Tuesday	Wednesday	Thursday
51,75	94	45,75	44	55,25	44,25
22,75	97	37,25	45,25	50,75	42,25
33,75	63	42	45,25	30,25	29
32,5	55,5	61,25	31,75	59	44,75
40,5	56,3	43,75	61	43,25	59,25